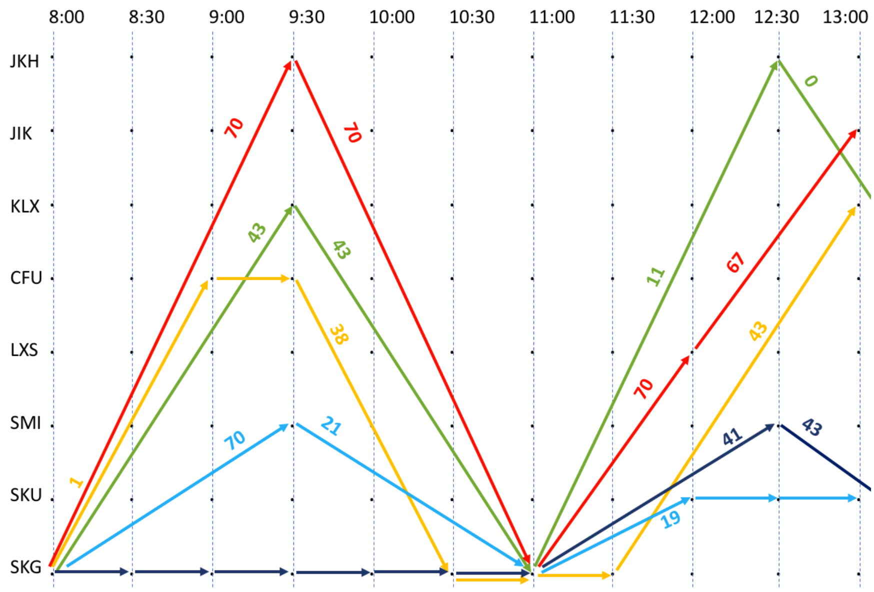

Optimization model to design aviation networks

Greek PSO case study

Francisco Costa Pereira Leandro

Thesis to obtain the Master of Science Degree in

Mechanical Engineering

Supervisors: Prof. António Ramos Andrade

Prof. José Maria Campos da Silva André

Examination Committee

Chairperson: Prof. Carlos Frederico Neves Bettencourt da Silva
Supervisor: Prof. António Ramos Andrade
Member of the Committee: Prof. Duarte Pedro Mata de Oliveira Valério

November 2019

Acknowledgments

I would like to thank my supervisor, Professor António Ramos Andrade for his availability from the first meeting, and assistance throughout the development of this thesis. Also, the help from Professor Sofia Kalakou was very important during the development of the predictive model, first with the gathering of data, and later with relentless assistance during the extensive “trial and error” process of defining the demand model. Finally, I must acknowledge the availability, support and insight provided by Professor António Pais Antunes, whose vast experience in this research subject certainly contributed to this thesis’s final result, complemented by the suggestions given by his PhD student, Sebastian Birolini, who is developing his research in a similar area. Also, I am thankful to Professor José Maria André for accepting to participate in this project.

Abstract

Aviation has been facing a constant expansion since the first flight by an airplane was achieved, and nowadays the success of aviation, is closely linked with the success of local economies. On the other hand, the airline business is a challenging environment to thrive economically, due to the high costs and low profit margins involved. Hence, it is critical that airlines optimize their operation, in order to succeed in the long term, in this highly competitive business. Several models have been proposed in the literature to support the optimization of airline fleets, with the objective of minimizing the operational cost for the airline. On the other hand, the amount of literature dedicated to the optimization of the usage of airline's fleets dedicated to Public Service Obligation (PSO) routes is much sparser. Based on the research developed by Pita et al. (2013), with case studies applied to the PSO networks of the Azores and Norway, this model is adapted, the objective of which is not only to minimize the direct cost to the airline, but also to minimize the total social costs. Then, the model is applied to a new case study based on a PSO network for the Greek islands, with key differences from the previous case studies such as strong competition from the ferry boat service.

Keywords: Public Service Obligation; Greek Islands; Flight Scheduling and Fleet Assignment; Integer Linear Programming; Optimization

Resumo

O negócio da aviação tem estado em constante expansão desde que se realizou o primeiro voo com um avião e, atualmente o sucesso da aviação está fortemente correlacionado com o sucesso das economias locais. Por outro lado, esta é uma área de negócio onde não é fácil ter sucesso devido aos elevados custos e reduzidas margens de lucro. Assim, é crucial que as empresas otimizem a sua operação, de forma a se manterem em operação no longo prazo, neste mercado altamente competitivo. Vários modelos foram publicados na literatura, com a finalidade de otimização dos recursos das empresas, com o objetivo de minimizarem os custos para estas. Por outro lado, é comparativamente reduzida a quantidade de publicações dedicadas à otimização de recursos de empresas dedicadas a redes de obrigações de serviço público. Baseado nas publicações por Pita et al. (2013), com casos de estudo aplicados às redes OSP nos Açores e Noruega, o modelo utilizado nestas publicações é adaptado. O objetivo destes modelos é, para além da redução do custo para as empresas, também a maximização da qualidade do serviço para os passageiros, e é aplicado a dois casos de estudo situados nas ilhas gregas, com características diferentes, nomeadamente a concorrência do transporte por barco.

Palavras-chave: Obrigações de Serviço Público; Ilhas Gregas; Programação de Voos e Atribuição de Frota; Programação Linear Inteira; Otimização

Table of Contents

Acknowledgments ii
Abstract iii
Resumo iv
Table of Contents v
List of Figures vii
List of Tables viii
List of Acronyms ix
1. Introduction 1
1.1. Context 1
1.1.1. The need for PSO Networks 1
1.1.2. PSO Framework 2
1.1.3. Integrated Flight Scheduling and Fleet Assignment Problem 4
1.2. Research Objectives and Methodology 5
1.3. Dissertation Outline 6
2. Theoretical Background and State of the Art 7
2.1. Theoretical Background 7
2.1.1. Demand Prediction 7
2.1.2. Mixed Integer Linear Programming 10
2.1.2.1. LP solving algorithm 10
2.1.2.2. Branch and Bound method 14
2.2. State of the Art 16
2.2.1. Demand Prediction 17
2.2.2. Fleet Optimization 20
3. Optimization model 23
3.1. Integer Linear Programming Formulation 23
3.1.1. Inputs 23
3.1.2. Pre-processed variables 24
3.1.3. Decision Variables 24
3.1.4. Objective function 25
3.1.5. Constraints 27
3.2. Illustrative example 30
3.2.1. Problem Specification 30
3.2.2. Results 31
4. Two Greek Case studies 34
4.1. Rhodes Network 37
4.1.1. Aircraft Operating Costs 37
4.1.2. Aircraft Ground Costs 38
4.1.3. Passenger Time Costs 38
4.1.4. Total Cost 39
4.2. Thessaloniki Network 39
4.2.1. Aircraft operating costs 40
4.2.2. Aircraft ground costs 41
4.2.3. Passenger time Costs 41
4.2.4. Total Cost 41
5. Predictive Model 42

5.1. Context 42
5.2. Definition of explanatory variables 42
5.3. Collection of data 43
5.4. Choice of model 45
5.5. Regression analysis 47
5.6. Final model 49
6. Analysis and Discussion 53
6.1. Application to Rhodes Network 53
6.2. Application to Thessaloniki Network 58
6.3. Exploring Scenarios 63
6.3.1. Rhodes Network 63
6.3.2. Thessaloniki network: 64
6.3.3. Different Scenarios 65
7. Conclusions and Future Research 66
7.1. Conclusions 66
7.2. Limitations 67
7.3. Future Work 67
8. References 69
9. Appendix 71
9.1. Plots of predicted vs real demand 71

List of Figures

Figure 1: Example of graphical presentation of the first criterion [23] 13
Figure 2: Example of graphical presentation of second and third criteria [23] 13
Figure 3: Demonstration of the Branch and Bound method [24]............. **Erro! Marcador não definido.**
Figure 4: Possible itineraries for a passenger...25
Figure 5: Graphical results from the illustrative example 32
Figure 6: Detailed results from the illustrative example 32
Figure 7: Demonstration of the capability to not exceed the maximum load factor...............................33
Figure 8: Graphical representation of the current Rhodes network. 37
Figure 9: Graphical representation of the Thessaloniki network 40
Figure 10: Comparison between expected and real values for model 2 on a linear scale....................50
Figure 11: Comparison between expected and real values for model 2 on a logarithmic scale 50
Figure 12: Comparison between expected and real values for model 5 on a linear scale....................51
Figure 13: Comparison between expected and real values for model 5 on a logarithmic scale 51
Figure 14: Key data provided by the optimization software, relative to the Rhodes network................54
Figure 15: Plots presenting the evolution of the objective versus time, for the Rhodes network 54
Figure 16: Extract of the text file exported by the software with the details of the optimal solution 56
Figure 17: Representation of the flights from 08:00 to 13:00 in the Rhodes network 57
Figure 18: Representation of the flights from 13:00 to 19:00 in the Rhodes network 57
Figure 19: Representation of the flights from 19:00 to 00:00 in the Rhodes network 58
Figure 20: Key data provided by the optimization software, relative to the Thessaloniki network 59
Figure 21: Plots presenting the evolution of the objective versus time, for the Thessaloniki network. .59
Figure 22: Extract of the text file exported by the software with the details of the optimal solution 60
Figure 23: Representation of the flights from 08:00 to 13:00 in the Thessaloniki network....................61
Figure 24: Representation of the flights from 13:00 to 19:00 in the Thessaloniki network....................62
Figure 25: Representation of the flights from 19:00 to 00:00 in the Thessaloniki network....................62
Figure 26: Comparison between expected and real values for model 1 on a linear scale....................71
Figure 27: Comparison between expected and real values for model 1 on a logarithmic scale 71
Figure 28: Comparison between expected and real values for model 3 on a linear scale....................72
Figure 29: Comparison between expected and real values for model 3 on a logarithmic scale 72
Figure 30: Comparison between expected and real values for model 4 on a linear scale....................73
Figure 31: Comparison between expected and real values for model 4 on a logarithmic scale 73

List of Tables

Table 1: Summary of relevant documents in demand prediction literature 20
Table 2: Summary of relevant documents in fleet optimization literature 22
Table 3: demand to be fulfilled in the illustrative example 30
Table 4: maximum load factor for the illustrative example 31
Table 5: Key data regarding the PSO scheme [10] 34
Table 6: Key data regarding the Greek PSO scheme (continuation) [10] 35
Table 7: Number of flights per O/D pair in the Rhodes network 38
Table 8: Travel time per O/D pair for the Rhodes network 39
Table 9: Number of flights per O/D pair in the Thessaloniki network 40
Table 10: Travel time per O/D pair for the Thessaloniki network 41
Table 11: Example of collected sociodemographic and economic data 44
Table 12: Example of collected transport market data 45
Table 13: Result of the Lagrange multiplier test 47
Table 14: Test of Model effects for the first regression 47
Table 15: Goodness-of-fit values for the first regression. 47
Table 16: KPI's of the different predictive model candidates 49
Table 17: Key for the p-values of the explanatory variables 49
Table 18: Demand values for each O/D pair for the Rhodes network 52
Table 19: Demand values for each O/D pair for the Thessaloniki network 52
Table 20: Key for the airport IATA codes in the Rhodes network 56
Table 21: Key for the IATA codes of the airports in the Thessaloniki network 61

List of Acronyms

PSO	Public Service Obligation
UAE	United Arab Emirates
FSFA	Flight scheduling and fleet assignment
IFSFA	Integrated flight scheduling and fleet assignment
EAS	Essential Air Services
RASS	Remote Air Service Subsidy
DoT	United States Department of Transportation
HCAA	Hellenic Civil Aviation Authority
KPI	Key performance indicator
GDP	Gross domestic Product
GZLM	Generalized linear model
GLM	General linear model
O/D	Origin-destination
AIC	Akaike’s information criterion
LL	Log-Likelihood
ILP	Integer Linear Programming
MILP	Mixed Integer Linear Programming
BB	Branch and bound
LP	Linear Programming
US	United States of America
EU	European Union
GB	Gigabyte
RAM	Random access memory
IATA	International air transport association

1. Introduction

Airports are key drivers of economic development for their respective catchment area([1], [2], [3]). This is even visible in a global context, in extreme cases such as Dubai, which is nowadays a thriving emirate of the United Arab Emirates (UAE). This happened after significant development and growth, due to the strong investment and clear strategy which led to investing in their airport and airline carrier to make it a global hub for airline transport [4].

Due to the close dependence between the performance of the aviation business and economies, it is very important to set conditions that will allow this business to thrive, with the associated economic benefits. Besides this, the airline business is known to operate in challenging conditions. This is explained by the high costs and low profit margins involved, making it crucial for an airline's long-term survival to use its resources in the most optimal way possible. There is extensive literature analyzing options to increase the operating margins of airlines (such as [5], [6], [7]).

Also, there is extensive literature addressing specifically the problem of optimizing the airlines' flight scheduling and fleet assignment (FSFA). The objective of such literature is to reduce the total operational cost for the airline, as this reduction contributes significantly to a better balance sheet of airline companies. The result is usually the suppression of frequencies in less profitable routes and allocation of the resources to the most profitable ones.

However, there are other routes, whose main objective is not to maximize profits but to provide accessibility to remote areas, where there is not enough demand for profitable airline operation. Nonetheless, this operation is vital for local communities, and in these networks, the objective is not only to minimize costs, but also to maximize the quality of the service provided to passengers. In this research area, there is comparatively less published literature, with an opportunity for important research.

1.1. Context

The Public Service Obligation (PSO) classification is a tool that can be used by the European states, in order to promote the economic development of regions which may require assistance, besides the normal self-regulation imposed by that market. This is achieved by subsidizing (with public money) the operation of airline routes in areas where such operation is not profitable.

1.1.1. The need for PSO Networks

In regions where, due to the low economic activity, scheduled airline service is not profitable but is necessary to allow for businesses to develop, and for residents to have accessibility to the important economic centers adjacent to their region, a PSO network can be imposed by the government. This grants a subsidy to the operation of an airline, in order to make it profitable to operate, at reasonable ticket prices. As the website of the European commission states: *"In order to maintain appropriate scheduled air services on routes which are vital for the economic development of the region they serve, Member States may impose public service obligations on these routes."* [8]. All the requirements and conditions to impose these networks are defined by the European commission's regulation No 1008/2008 [9].

Currently there are over 170 PSO networks in operation throughout Europe [10] mostly composed of domestic routes distributed by 8 EU countries, plus Iceland and Norway. The most obvious regions where this may be required, and which accounts for the majority of PSO networks in force is in islands. Due to its geographical characteristics, they are prone to requiring subsidies for scheduled airline service to be profitable. Nevertheless, there are also several PSO networks, which do not involve islands, but require such networks to connect remote areas with adjacent economic centers.

As such, the imposition scheduled airline services was used by the legislators of the European Commission as a tool to promote the development of smaller, less populated areas. This scheme has already obtained verifiable positive results since the initial implementation, such as stronger connectivity to important centers for the residents and development of tourism in the remote region where a PSO service was implemented [11].

1.1.2. PSO Framework

PSO networks are exceptions to the free competitive market which is operated throughout Europe since the policy of deregulation started to be implemented in 1988. These networks have guidelines defining their base principles and defining how a new PSO network should be imposed.

The basic principles that support the European PSO system are [12]:

1. Transparency: all calls for tenders, awards, modification and abolition of PSO routes must be announced in the official journal of the EU. Also, airfares and conditions can be quoted to users;
2. Market failure: PSO routes are only imposed after market forces have failed to make scheduled air service profitable on the route;
3. No obstacle to market functioning: a PSO should not limit the possibility for air carriers to provide a higher level of service (regarding frequency and capacity), than the minimum obligations required under the PSO;
4. Necessity: Routes are considered vital for the economic and social development of the region served (routes to an airport serving a peripheral or development region or thin routes to any airport).

5. Proportionality and non-discrimination: PSOs must be imposed in a proportionate and non-discriminatory manner (e.g. no restrictions based on passenger's nationality or on the air carrier's state of origin, no selective promotion of specific air carriers/airports);
6. No alternative: Inadequacy of alternative transport modes connecting the route(s) under PSO;
7. EU law: Full compliance with EU Regulation 1008/2008 (compliance with national law only is insufficient).
8. Route-by-route basis: Necessity of PSO award must be assessed for each route separately (no network routes). A PSO cannot link two cities or two regions, routes must be defined from airport to airport.
9. Geographic scope: A PSO route between an EU airport and a non-EU (except EEA members) country is not allowed. Intra-EU routes (not exclusively domestic) are however allowed.

There are two types of PSOs:

- **Open PSO** (22.1% of total routes as of July 2019): any air carrier can operate the PSO if it complies with their requirements; no exclusivity; no compensation granted.
- **Restricted PSO** (77.9% of total routes as of July 2019): in case no air carrier is interested in operating the route on which the obligations have been imposed, the state concerned may restrict the access to the route to a single air carrier and compensate its operational losses resulting from the PSO. The selection of the operator must be made by public tender at Community level: only one air carrier can operate the PSO; if exclusivity is not enough to ensure the financial viability of service, then compensation is awarded.

Although point 8 of the PSO principles defines a route-by-route basis, airlines may apply for a group of PSO routes, and be awarded exclusive operation to this group of routes, should this result in an increase in operational and administrative efficiency. This is what happens, for example, in the Azores Archipelago (in Portugal), where all the inter-island flights are operated by SATA Air Açores.

Restricted PSOs have stricter rules, such as:

- tender is open to EU or EEA carriers only;
- Air carrier selection as soon as possible;
- contract awarded for four years (five years for outermost regions);
- bid selection criteria: adequacy of service offered and level of compensation;
- compensation level must not exceed the amount required to cover the net costs incurred.

The process of imposing a PSO is started by regional or national governments, and begins with an invitation to tender, which must be published in the Official Journal of the European Commission. The tender usually stipulates minimum service levels and maximum fares that air carriers need to satisfy for the duration of the contract. There are two tender rounds. The initial tender asks for submissions from air carriers who are able to operate services and meet the tender specifications without subsidy. If no carrier is willing to offer a subsidy-free operation, a second tender is issued which invites carriers to bid based on receiving a subsidy. The awarding authority then decides considering the level of subvention demanded, levels of service offered and any other relevant considerations [13].

The definition of the PSO implies a minimum level of service to be defined, usually either by a minimum number of annual seats, or a minimum number of weekly frequencies. This minimum level of service is publicly available in the EC's website [10].

These networks also exist in the United States of America, though applied slightly differently, in what is called the Essential Air Services (EAS) system [14], or in Australia with the Remote Air Service Subsidy (RASS) [15].

Although these three subsidy programs have the same overall purpose, they were implemented in different ways, with different guidelines. For example, while the PSO system allows for national or regional governments to setup PSO networks, in the American EAS, it is the responsibility of the Department of Transportation (DoT) to set them up, in a more centralized decision process. Moreover, while the PSO system awards exclusivity of the operation to the carrier which wins the tender, for the period of the contract, the EAS system allows for other carriers to enter that route whenever they want, even though one carrier may have won the tender already. Then the awarded carrier has to decide if it wants to continue operating without subsidies, or to withdraw from the route [13].

Even inside the PSO system within Europe, though under the same subsidy framework, each country has used it in different ways (as noted in [13] and [16]). Some countries use the PSO scheme in sparse regions of the country, to link remote regions between themselves (as is the example of Norway), while France, on the other hand, uses the PSO scheme to connect smaller regions with Paris, in an effort to promote economic activity in the remote regions, by connecting them to the business centers of the country [17].

Another example of different adoption of the PSO scheme is given by the United Kingdom, which focused its PSO network on setting up mostly "lifeline" types of services, allowing the population of remote regions to reach a city with more facilities (such as hospitals, for example) whenever necessary, not focusing on setting up routes for economic development.

1.1.3. Integrated Flight Scheduling and Fleet Assignment Problem

As stated above, the airline business operates in an economically and operationally challenging environment, with high costs and comparatively low profit margins. Although the airlines that operate within PSO networks are financially rewarded for their service, it remains critical for them to operate as efficiently as possible, in order to maximize economic results. On the other hand, the entity responsible for subsidizing the PSO network (usually national or regional government entities) is focused on maximizing the quality of service provided to the users, but is also interested in minimizing the cost of subsidies that it must provide to the airline operating the route. Hence, solving the Integrated Flight Scheduling Fleet Assignment problem (IFSFA), is the suitable tool to optimize such networks.

The IFSFA model builds on the FSFA (which is only focused in cost reduction to the airline) by adding the minimization of the costs associated with the passenger (from a financial and quality of service perspective), allowing for a more holistic view of the concept of an optimal network, in which the goal is not only reducing costs, but at the same time maximizing the quality of the service provided. This is very important because these networks are, by definition, sub-optimal, due to the fact that flights are being imposed on routes that do not have enough demand to justify their operation. Moreover, if a normal FSFA model was applied to these networks, the result would be a significant reduction in the quality of service provided, which might even violate the minimum frequencies defined by the entity which imposed the PSO, in an attempt to reduce the cost for the airline.

This problem has already been explored previously and published in the literature ([18] and [19]) with positive results, reducing both financial costs for the airline and time costs for the passengers. In the present dissertation, it will be applied to a different network with key differences, such as: the significant seasonality effect of demand (increasing in summer months due to tourism), or the significant competition from the ferry boat service (an established transport in the region).

1.2. Research Objectives and Methodology

The purpose of this research is to build on the model developed in [18] and adapt it in order to apply it to part of the PSO network of the Greek islands.

Therefore, initially the model developed by the above-mentioned research will be implemented using the Fico Xpress software package, and tested through an illustrative example, which will be used to verify the results and demonstrate the capabilities of the model. After this process, the model will be applied to two case studies, both part of the Greek PSO network, using data provided by the Hellenic Civil Aviation Authority (HCAA) and by Aegean airlines. The model will be further developed to account for the specific characteristics of the PSO network of the Greek islands, such as the seasonality effects or the specific restrictions of smaller Greek airports.

The current network will be analyzed and its respective costs will be calculated, which serve as the basis of comparison for the performance of the optimization model. Then, after solving the optimization problem, the results will be discussed and compared with the current network design.

Finally, possible improvements to the network will be presented and discussed, and the research limitations and further research opportunities will be discussed.

1.3. Dissertation Outline

This document is structured in seven chapters.

Chapter 1 provides an introduction to the subject of the dissertation, stressing the importance of the overall aviation business, and detailing the specific aspects and objectives of the PSO network, comparing it to its similar frameworks outside the EU, followed by the explanation of the relevance of optimizing these networks. Finally, the research objectives and methodology are presented, followed by the general presentation of the structure of the dissertation.

Chapter 2 builds on the previous chapter, with a goal of contextualization of the research topic, this time explaining the fundamental theoretical concepts behind integer programming optimization algorithms. This is followed by a State of the Art review, where the most prominent scientific papers in this research area will be reviewed and discussed, setting the theoretical foundation for the research that will follow.

Chapter 3 presents the model to be used in the optimization, detailing the objective function and the constraints implemented. In this chapter, the mathematical formulation of the model is presented and described. An illustrative example is presented and serves the purpose of model verification.

Chapter 4 is where the two case studies are presented. The networks are discussed, and the similarities and differences between them are explored. Moreover, the costs associated with each network are detailed, and the underlying assumptions are explained.

Chapter 5 deals with the predictive model. A comprehensive description of the complete process is presented, while discussing the challenges and important aspects associated with this task. The data collection process is introduced, followed by the different approaches adopted until reaching the final model. Each attempt is presented with the value of the associated key performance indicators (KPI), in order to allow the reader to better understand the evolution until the final model is obtained. This quantifies the quality of the model, and demonstrates the improvements during the selection process.

Chapter 6 presents the results obtained by applying the optimization model to the two case studies, detailing the costs of the optimized networks, and their key features. It also compares the optimal solution with the current networks defined and quantified in chapter 4, proposing an explanation for the improvements or possible deteriorations in the optimized solution, compared to the current situation.

Finally, **chapter 7** reviews the work done throughout the dissertation, comparing the obtained results with the initial objective. The main conclusions of the research are presented, as long as its limitations, identifying further research opportunities, as a continuation of this work.

2. Theoretical Background and State of the Art

2.1. Theoretical Background

The present research focuses on two main subjects, which are relevant in current scientific research:

1. Demand Prediction;
2. Integrated Flight Scheduling and Fleet Assignment.

Both these subjects have consistently attracted attention from the scientific research community, due to their importance in a wide set of applications, with a key focus of interest from the transportation sector. In the following chapters, the theoretical foundation for both these subjects will be presented, in order to provide context to the research that follows.

2.1.1. Demand Prediction

The research area of demand prediction is most commonly based on multiple variable regression models, due to the fact that such a complex task requires several variables in order to make the results as accurate as possible. There are several possible formulations, starting with the most simple case of linear regression, where it is assumed that the dependent variable (which is the variable to be predicted, in this case the demand between origin and destination) varies linearly with several explanatory variables (e.g. the population of origin and destination, the gross domestic product (GDP) per capita, etc). This can be represented by the general expression:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_i x_i + \epsilon \qquad (1)$$

Where y is the dependent variable, x_i are the explanatory variables, β_0 is the y-intercept and β_i are the coefficients for each explanatory variable.

The biggest advantage associated with this type of regression is its simple form, allowing for the regression to be performed with simpler calculations, but it is obviously more limited and is not suitable to more complex problems.

For applications where a linear dependence on the explanatory variables is not suitable, there is a wide variety of more adequate models to choose from, each model with its strengths and weaknesses. These models are part of the so called Generalized Linear Models (GZLM) group, which opens the possibility of performing regressions in datasets where:

- The dependent variable may not be continuous;
- The effect of the explanatory variables may not be linear.

Generalized Linear Models should not be confused with General Linear Models (GLM), because the latter are applicable to normally distributed dependent variables, whilst GZLM allows for dependent variables with non-normal distributions. This chapter will focus on the GZLM type of models which are relevant for the present research.

A GZLM has 3 components:

1. The systematic component: Usually a linear predictor such as $\eta_i = \beta.X$, where X is a vector of explanatory variables, β are the calibration parameters and η_i is the link function;
2. A link Function: Describes how the mean, $E[Y_i] = \mu_i$, depends on the linear predictor $g(\mu_i) = g(E[Y_i]) = \eta_i$ and, inversely, $\mu_i = g^{-1}(\eta_i)$;
3. A random component: Usually a probability distribution from the exponential family.

There are several available GZLM (Normal, Poisson, Gama, Inverse Gaussian, etc.) but the types of model most suitable for this problem are the Poisson distribution or the Negative Binomial, since both are applied to similar cases, but the choice depends on the dataset, not on the type of problem. This is due to the fact that this type of regression is preferred when events are rare, and there are significant differences in the available dataset of dependent variables (which are used to tune the model's coefficients in order to then predict the remaining dependent variables).

This is applicable to the present research, because in PSO networks the demand is low (compared to normal, profitable airline routes) and there is significant discrepancies in the values of demand for different origin-destination (O/D) pairs. In fact, as it is explained later on, in this case study's data there were demand values of around 10 and others around 5500, i.e. 550 times higher.

While the Binomial distribution is used when the dependent variable corresponds to data counts of successes per number of trials (which would not make sense in this situation), the Poisson distribution is used when the dependent variable represents successes (in this case it represents the number of passengers for that O/D pair) per given number of time units (in this case for one month).

One of the criteria to use a Poisson distribution is that there is no overdispersion of data in the dataset (i.e. the variance and the mean are equal to the lamda parameter of the distribution). In datasets where there is overdispersion, which usually happens when there are too many zeros in the dataset, corresponding to the demand in this type of routes, the alternative to the Poisson distribution is the Negative Binomial distribution. Hurdle models are also increasingly popular nowadays for applications of this kind, with a high quantity of zeros in the dataset, relative to the overall amount of available data [20].

In case the chosen link function is logarithmic, the Poisson regression has the following general expression:

$$E[Y_i] = \lambda_i = \exp\,(\beta_0 + \sum_{j=1}^{p} x_{ij}\beta_j) \tag{2}$$

Where $E[Y_i] = \lambda_i$ represents the expected value for the variable being estimated, β_0 and β_j represent the coefficients of the regression and x_{ij} represent the values of the explanatory variables.

On the other hand, the Negative Binomial regression has the following general expression:

$$E[Y_i] = \lambda_i = \exp\,(\beta_0 + \sum_{j=1}^{p} x_{ij}\beta_j + \varepsilon_i) \tag{3}$$

The only difference between these two general expressions is the error term ε_i, which adds additional variance, and thus is included due to the overdispersion of data. Hence, the Poisson model can be regarded as a limited model of the negative binomial where the variance of ε_i set to 0.

To perform the regression, the first step is checking if there is overdispersion of data, which can be achieved through the Lagrange multiplier test. A non-significant Lagrange test coefficient indicates that the binomial model's ancillary (dispersion) parameter cannot be assumed to be different from 0. If this is the case, a Poisson model is preferred over a Negative Binomial model.

With the regression model selected, the following step is of a trial and error nature, starting with attempting the regression with no explanatory variables. This is designated as the unrestricted model, and will be used as a basis for comparison. Then, through trial and error, possible explanatory variables are added to the regression successively, being kept if their effect is significant and the sign of the coefficient makes sense, or removed otherwise, until the best performing model is achieved.

There are several indicators which can be used to compare models between themselves and choose the one which performs best, such as:

- Akaike's information criterion (AIC) and deviance, where smaller values indicate better performance of the regression (for both indicators);
- Log-Likelihood (LL), where greater values indicate better performance;
- Pseudo R^2, obtained by comparing the LL of the model being tested with the LL of the unrestricted model, where values closer to 1 indicate better performance.

For these types of regressions, there is still a tool that may be used for datasets with significant differences in the values of the dependent variable (this is the case of this dataset, where demand values range from 10 to 5500). This tool is called the offset variable, which is by itself an explanatory variable, and the coefficient of which is fixed at 1 and is thus constant throughout the fitting process. Then, depending on the binary value for the specific entry (in this case it represents the demand for a specific route), it will either sum one (for high demand) or zero (for low demand) to the value in the exponential.

The information above was only a brief review of the theoretical concepts, and was based on the book by McCullagh and Nelder [21], and the book by Silva and Turkman [22]. For further research, these references are recommended.

2.1.2. Mixed Integer Linear Programming

The problem that is explored in this dissertation is formulated using an Integer Linear Programming (ILP) model. This is part of the broader set of problems, defined as Mixed Integer Linear Programming (MILP) problems. Such problems are characterized by having necessarily at least one decision variable restricted to be an integer and also the objective function defined by a linear equation and the constraints by linear inequalities. ILP problems have all decision variables restricted to have integer values.

There are several methods for solving MILP problems, but in this section only the branch and bound (BB) method will be presented, which is commonly used by commercial software. It is characterized by being an efficient approach, which instead of searching in the whole range of possible, leading to the consequence of unreasonable computational times, it only searches in the minimum number of solutions possible, in order to accelerate the process and increase computational efficiency.

The methodology applied to solve MILP problems involves two key steps. The first step is called linear programming (LP) relaxation, and corresponds to eliminating the restriction that imposes the decision variable(s) to have integer value(s) (hence the term relaxation), and solve the associated problem. This sets the target for the MILP problem (which, being more restricted will, at best, reach the same solution, but never a better one) to be solved. By using such target as its basis, the BB method is applied, to find the optimal integer solution.

2.1.2.1. LP solving algorithm

As explained above, although this dissertation does not explicitly deal with LP problems, solving them is the first step in the methodology to solve a MILP, and for that reason this sub-chapter will briefly discuss this subject, not as an in-depth analysis, but just setting a theoretical background on the concepts involved with solving these problems.

Probably the method for solving LP problems with most widespread acceptance is the simplex method, because of its robustness and readiness to implement in computer algorithms, as it is self-initiating. It also has the additional benefit of, besides calculating the optimal solution, indicating how the optimal solution varies as a function of the problem data. This method has been extensively discussed in literature ([23], [24]).

The simplex method gives immediately the optimal solution when the objective function and constraints of the problem are written in the canonical form. Hence, the procedure to determine the optimal solution through the simplex method consists in manipulating the equations of the problem until it is written in the canonical form, characterized by:

- All decision variables are constrained to be non-negative;
- All constraints, except for the non-negativity of decision variables, are stated as equalities (when they are defined as inequalities they can be easily converted to equalities through the addition of "slack variables");
- The righthand side coefficients are all non-negative;
- One decision variable is isolated in each constraint with a positive coefficient of one;
- The variable isolated in a given constraint does not appear in any other constraint, and appears with a zero coefficient in the objective function.

Once the equations have been manipulated to reach a form that complies with all these requirements, the optimal solution can immediately be determined. In general, given a canonical form for any linear program, a basic feasible solution is given by setting the variables isolated in each constraint, which are called the basic-variables, equal to the righthand side of the corresponding constraint and by setting the remaining variables, called non-basic variables, all to zero. From the previous statement the optimality criterion is defined:

- In a maximization problem, if every non-basic variable has a non-positive coefficient in the objective function of a canonical form, then the basic feasible solution given by the canonical form maximizes the objective function in the feasible region.

On the other hand, the problem may be unbounded over the feasible region, that is, not having a real number as its maximum, which tends to infinity. This can be defined by the unboundedness criterion:

- In a maximization problem, if any non-basic variable has a positive coefficient in the objective function of a canonical form, and has negative or zero coefficients in all constraints, then the objective function is unbounded from above over the feasible region (that is, the maximum tends to infinity).

The only situation that may appear with problems in the canonical form but is not contained in the two criteria above, is the situation where one non-basic variable has a positive coefficient in the objective function of a canonical form, but also has a positive coefficient in at least one of the constraints. In these situations, although the current set of equations complies with the requirements of the canonical form, these equations can be manipulated in order to reach a situation which will fit in the optimality criterion. This can be achieved by replacing the non-basic variable which has the positive coefficient in the objective function with the basic variable that has the lowest value of $(\frac{\beta_1}{\beta_2})$, being β_1 the coefficient that is multiplied by that basic variable in the constraint where it appears (there is only one basic variable per constraint, in the canonical form), and β_2 the coefficient that is multiplied by the non-basic variable in the same constraint.

This replacement will make the non-basic variable become a basic-variable, and vice versa, and can be achieved through algebraic manipulation, in a process known as pivoting. Once the expression was pivoted, it will now comply with the requirements of the optimality criterion and the solution will be immediately obtained. The criterion for this third scenario can be defined as the improvement criterion:

- In a maximization problem, if any non-basic variable has a positive coefficient in the objective function of a canonical form, and a positive coefficient in at least one constraint, then a new basic feasible solution may be obtained by pivoting.

From [23], a graphical representation of the three criteria was presented, for problems with two non-basic variables. These three scenarios (each applicable to one of the above criteria) were restricted to the same constraints, which define the feasible region:

$$x_1 - 3x_3 + 3x_4 = 6 \tag{4}$$

$$x_2 - 8x_3 + 4x_4 = 4 \tag{5}$$

$$x_j \geq 0 \ (j = 1,2,3,4) \tag{6}$$

And each one will have a different objective function defined:

1. Optimality criterion: $Max\ Z = -x_3 - x_4 + 20$ (7)
2. Unboundedness criterion: $Max\ Z = 3x_3 - x_4 + 20$ (8)
3. Improvement criterion: $Max\ Z = -3x_3 + x_4 + 20$ (9)

It can be easily verified that each of the three problems defined above complies with the condition of the associated criterion, and Bradley et al. [23] provided graphical representations of the feasible region for the associated constraints [(4),(5),(6)] as presented in Figures 1 and 2. Here, the limits defined by the three constraints and the optimum solution for each problem, are represented. In Figure 1, the evolution of the objective function (increasing its value from 17 to 20) until reaching the optimum point defined by (0;0) on the non-basic variables x_3 and x_4 is also visible.

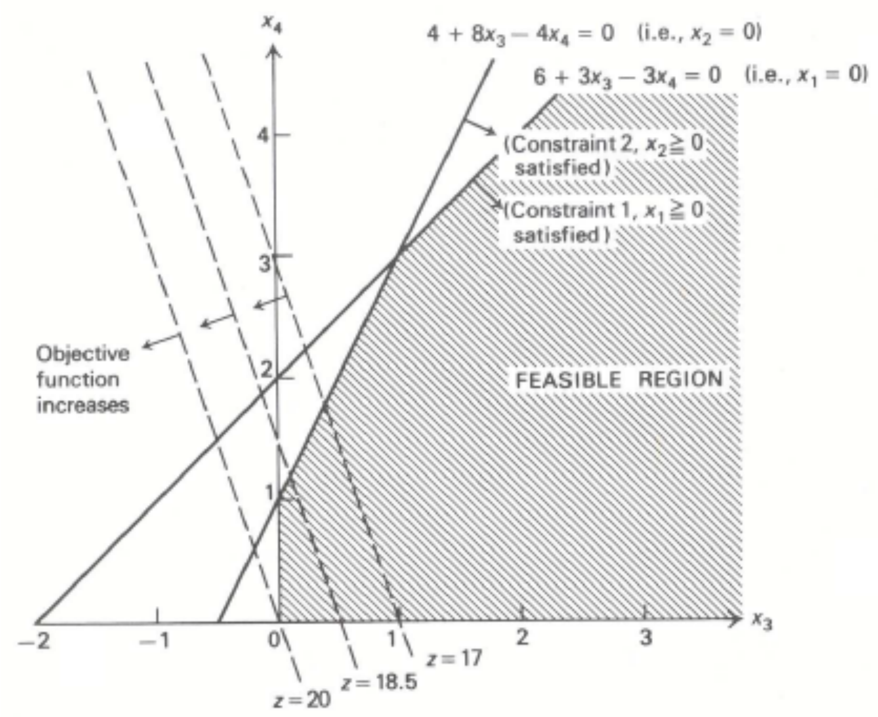

Figure 1: Example of graphical presentation of the first criterion [23]

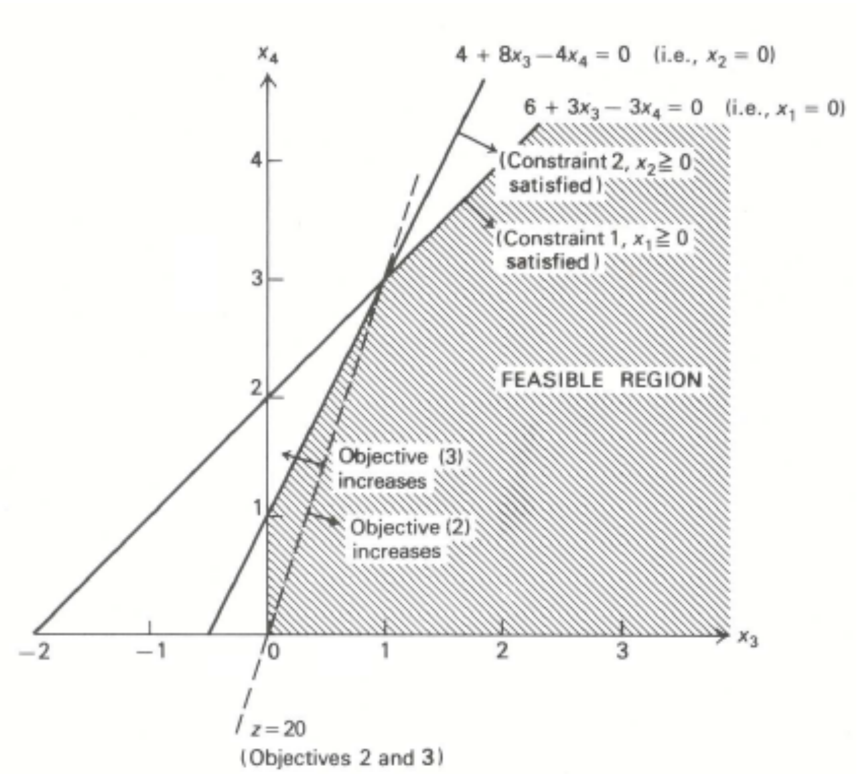

Figure 2: Example of graphical presentation of second and third criteria [23]

Figure 2 represents once again the feasible region and the evolution of the objective function of the second and third criteria, the first one increasing towards infinity and the second one towards its maximum at (0;1), which is explained by the fact that x_4 was replaced by x_2 as a non-basic variable, and became a basic variable (hence its value of one instead of zero).

2.1.2.2. Branch and Bound method

The first step in the branch and bound method is performing the linear programming relaxation. The solution will provide the upper bound for the solution in a maximization problem, or the lower bound for the solution in a minimization problem, i.e. the best possible result achievable. If, by chance, this solution obeyed the restriction of the decision variable(s) to have integer value(s), this would immediately give the optimal solution. Naturally, the probability of this happening is low and the model will usually have to continue to the next step.

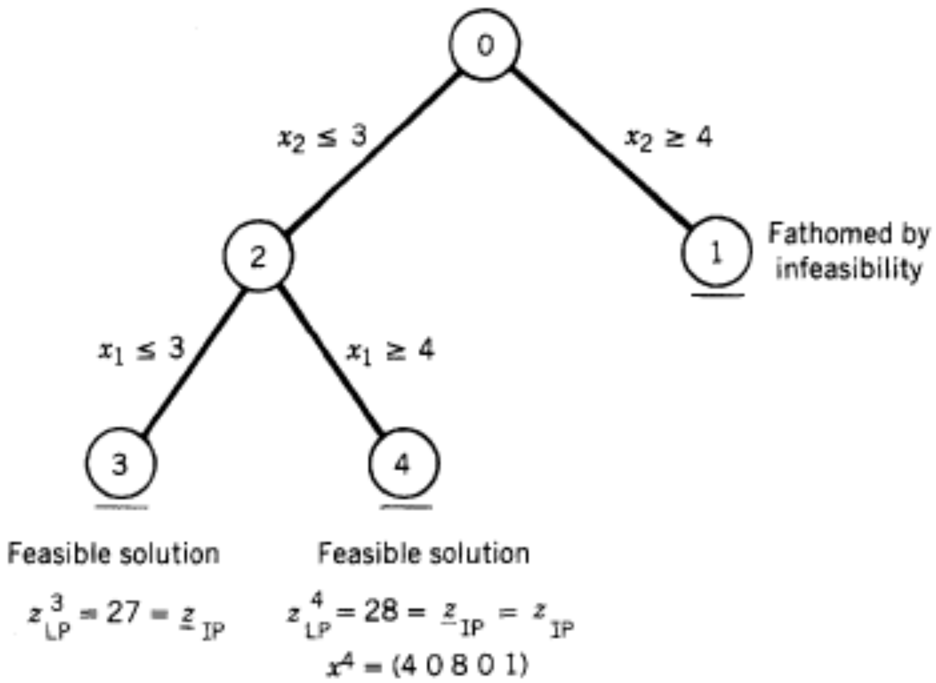

Figure 3: Demonstration of the Branch and Bound method [24]

Assuming the solution of the LP relaxation does not obey the restrictions of the problem, this solution will be the first "node" of the BB tree (node 0 in Figure 3). From this node, branches will lead to new nodes (nodes 1 and 2 in Figure 3), increasing what is defined as depth. These two branches will result in 2 new nodes, with a depth increased by one unit. Hence, the first node has an associated depth of 1, the two nodes that result from this have a depth of 2, and so on. These new nodes, will have an additional constraint that imposes that the value of one of the decision variables must be either <=x (in one node), or >=(x+1) (in the other node), assuming this decision variable had a value between x and (x+1), and must be an integer in the final solution. In the example of Figure 3, the chosen decision variable (x_2) had a value between 3 and 4, hence, node 2 had the additional constraint of ($x_2 \leq 3$) and node 1 had the additional constraint of ($x_2 \geq 4$).

Now, for each of the two nodes, which have an additional constraint to be satisfied (that imposes that the chosen decision variable is either "<=x" or ">=(x+1)"), the optimal solution will be calculated. The result is the lower bound in a maximization problem (or the upper bound in a minimization problem).

One important fact is that the final optimal solution will always be between the lower and upper bounds. This is important because in the first step of solving the problem (the LP relaxation), the lower bound is calculated (for a minimization problem) and every feasible solution obtained by the solver can be the optimal solution to the problem (at this stage, the upper bound is infinite). This implies a very large range of possible solutions, but every time the solver finds a new feasible solution, its value will be compared with the current upper bound. If the value of this solution is not within the interval defined by the lower and upper bounds (whose size will have the tendency to reduce with the progress of the optimization), it can be immediately discarded. This reduces the range of possible solutions and makes it possible for the solution finding process to be more efficient and, consequently, quicker.

After calculating the solution for both nodes, the bounds resulting from the two nodes will be compared and the one that has the lowest value (in a minimization problem), is the one which performed best and will be the focus of the solver.

At that node, if the solution has an integer value in the decision variable which was involved in the new constraint, two new branches will be formed from this node (nodes 3 and 4 in Figure 3). These new branches will impose that a different decision variable (x_1), which must have an integer value in the final solution, must either be "<=y" or ">=(y+1)", assuming this decision variable had a value between y and (y+1). In Figure 3, x_1 had a value between 3 and 4, hence node 3 had the additional constraint ($x_1 \leq 3$) and node 4 the additional constraint ($x_1 \geq 4$). This is an iterative procedure, that can easily be programmed by a computer, hence its applicability in commercial software. The model will increase the depth, until reaching the last possible node in that branch, where the branch will be fathomed (which can happen for 3 reasons, to be discussed later). This iterative process would increase depth (imposing more constraints) successively until either obtaining an unfeasible solution or a solution which obeys all restrictions imposed by the main problem to be solved.

In summary, each iteration involves the following operations:

1. Branching - deciding on which node to branch from among the "active" nodes in the tree;
2. Bounding - operation that is performed at each created node to decide which nodes will be fathomed (pruning the branches). It amounts to solving the corresponding subproblem at each node;
3. Fathoming - in essence pruning the tree. Based on some test and the bound values at the "active" nodes, the algorithm decides on nodes that can be discarded. This step also includes a termination criterion.

As mentioned above, fathoming is the process through which it is possible to stop the iterative process in that branch, and leads to a more efficient approach, allowing for less nodes to be examined overall. The fathoming process is guided by 3 criteria that decide if that node should be discarded or chosen as the best current solution. These 3 criteria will be explained below, being applicable to a minimization problem:

1. Fathoming by infeasibility: if the node being examined resulted in an unfeasible solution, any possible branch that could result from this would necessarily be unfeasible (by adding constraints, the range of solutions will never expand), hence the node is fathomed and abandoned;
2. Fathoming by bound: If the node being examined has a value greater than the current best value (which is in another node). In this situation, neither its solution or the possible solutions obtained by branching from this node would obtain better results than the current best value, hence it would simply be a waste of computational capability;
3. Fathoming by feasibility: If two nodes with feasible solutions branch from the same node, the one which has the highest value will be discarded, and the one which has the lowest value (which is the goal in minimization) will be chosen as the current best solution. There is no need to branch from this node as it is already feasible (hence obeys all the restrictions).

One important KPI allowed by the BB method is the optimality gap. This results from the already discussed fact that the optimal solution has to be confined within the interval defined by the lower and upper bound. Hence, it is possible during the optimization process to estimate how close the current best solution is from optimality. This KPI can be used to benchmark the quality of a still unfinished optimization process, by giving the difference in percentage between the current best solution that has been found and the lower bound (for a minimization problem, upper for maximization). This can be used as the criterion to stop the optimization at a reasonable value, if it is not possible or acceptable to wait until the optimum value is achieved. This KPI is designated as optimality gap and it can be calculated through:

$$Optimality\ gap\ (\%) = \frac{current\ best\ solution - lower\ bound}{current\ best\ solution} \times 100\ \% \tag{10}$$

2.2. State of the Art

As mentioned previously in the theoretical background chapter, this dissertation focuses on two key research subjects: demand prediction and fleet optimization. Both areas are currently at the focus of transportation research and, consequently, have been extensively discussed in published literature. Hence, this chapter is dedicated to reviewing the state of the art in these two subjects, setting a context for the research that follows.

2.2.1. Demand Prediction

This research subject has attracted considerable attention from the academic community, due to, at the same time, its importance and the difficulty in obtaining reliable estimates [25]. In this chapter, this literature will be analyzed and discussed.

Probably the most established method of demand prediction is the multiple variable regression analysis. Washington et al.'s book [26] provides a comprehensive analysis of the concepts related with this exercise, detailing the different models available and the scenarios where each model is applicable.

In 1997, Calderon [27] published a paper proposing a demand model for scheduled airline services, based on data from the entire European network in 1989. This research was innovative at the time, because the author identified a lack of literature dedicated to demand prediction in Europe, proceeding to adapt models developed in the United States of America (US), to be applicable to the European market. The author came up with gathered several significant conclusions that nowadays are widely accepted in specialized literature, such as the importance of population, GDP and frequency of flights as explanatory variables. The paper also performed an elasticity analysis of the resulting explanatory variables, proceeding then to propose explanations for the results obtained, characterizing the different segments of the European market.

Grosche et al. [28] proposed two possible gravity models, underlining the known fact that there is considerable unreliability with these models. This leads airlines to not relying solely on one model but gathering the information from different models to predict the demand for their possible future routes. It is applicable to new markets, with the advantage of not relying on traditionally used inputs that are not yet available to airlines before starting to operate the route (e.g. service-related factors or passenger income). Instead, the model uses mainly geo-economic variables as independent factors.

Wadud [29] published a paper analyzing demand prediction in areas where data to define explanatory variables is not available. This is applicable to, for example, a region where a study is carried out, assessing the importance of building an airport to the region. Hence, the model developed uses limited aggregate information about a country or region in order to generate a forecast for passenger demand of a new airport. This modeling approach was applied to forecast the demand for a new airport in a divisional capital in Bangladesh, for which no regional data on gross domestic product or population was available. This research highlights the challenges and uncertainties associated with the demand prediction task and performs it with limited data available. The model also takes into account the competition by road travel, which can be important in relatively small countries.

Focused on more commonly analyzed markets, Barnhart et al. [30] analyzed how to improve reliability in the air transportation segment in the US and European Union (EU) through managing capacity and demand in already established networks. The result proposes changes in several areas, such as tactical adjustments and real-time interventions, from medium to short term. This stresses the importance of technological progress in these improvements, in order to have a more “flexible” network which will be able to withstand the challenges that will be imposed in the future. This will be crucial with the continuous expansion of already saturated areas, such as airports (on the ground and in the surrounding airspace).

Demirsoy published a master thesis in 2012 [31] focusing on studying the significant expansion of the Turkish airline transport market, proposing and testing six hypothesis to explain such growth. The research concludes that, unlike what most published literature finds for other markets, in the Turkish market the population does not have an effect on demand in the long term (although it has in the short term). Another interesting feature is the analysis of the effect of deregulation (in 2003) in the demand for the Turkish market, which was significant. It also concluded that the high-speed railway network is not affecting demand for the Turkish air transport market in the short term, although it may affect in the long term, when its network is more mature. This is interesting for the present research, because it is also expected that the competition of ferry boat services will affect demand in its case studies. The differences in the results of Demirsoy’s case study from most published literature demonstrate that in such a complex subject, it is not possible to use solutions which were applicable to other markets immediately, but a thorough analysis must be performed to that specific market.

Adeniran & Adeniran [32] focuses on determining the correlation between international air travel demand in Nigeria and several econometric indicators. It is interesting to verify that the results were not what was expected by the authors, due to problems related with correlation and multicollinearity of the independent variables. This stresses the fact that, although an attempt to add as much independent variables as possible might seem a good method to increase the quality of the forecast, this is usually not the case in these regressions. Hence, attention must be paid when performing the regression, following the steps already described in the theoretical background, in order to maximize the predictive capacity of the model. Another interesting feature of this research is demonstrating the significant impact that variation in the value of local currencies compared to the US dollar may have. This is especially important in currencies with a smaller base of users, relative to, for example, the Euro which although still affected, is more stable.

Carmona-Benítez et al. [33] proposed an econometric dynamic model to estimate passenger demand, applying it to the Mexican market and proposing an approach to solve the airline airport hub location problem. This paper highlights the economic importance of the airline business to cities, by using economic indicators as explanatory variables for the model. It concludes that the increase of economic activity promotes air travel demand, and that economic indicators can be used correctly as explanatory variables to predict demand, at the state, city or airport level. This model is validated by two different tests, proving its suitability to predict demand for air travel.

Kluge et al. [34] perform an analysis applied specifically to the European market, hence its interest to this research. Their results can be analyzed in order to predict particular features of the European market, and later to compare the particularities of the Greek market, from the results of the this research. The paper focuses on determining the relation between passenger air travel demand and factors such as the GDP, the urbanization level, the geographical location and the degree of education, proving that the first, third and fourth indicators were statistically significant. The GDP of a country is commonly seen as a statistically significant variable in this area of research, but the other two indicators also deliver interesting results, and may have an effect on the Greek case study.

An interesting niche of this research subject is the demand prediction for markets with very strong touristic activity, due to the differences that these carry with them (such as seasonality, less importance of GDP when compared to more traditional business markets, very high ratios of tourist to inhabitant, etc). This type of markets have already been analyzed since at least 2002, when Devoto et al. [35] published their research focused on determining how demand could be predicted in these touristic markets, specifically using tourism variables (e.g. resident population, number of tourist beds, per capita beds and tourist arrivals). This was achieved through the application to a case study in Sardinia, Italy, analyzing three different airports, with the resident population always being statistically significant as a predictor of demand. Besides the city population adjacent to the three airports, there was a different variable related to tourism which was considered significant for each airport, demonstrating that the effect of tourism cannot be ignored and indicating that there is no consensus on which variable is more suitable. This happens even within three airports contained within a relatively short area, in a similar market. More recently, Erjongmanee & Kongsamutr [36] published a research focusing on demand forecasting in Thailand, taking into account the effect of tourism, with significant results as predicted. This paper also studies and compares the suitability of machine learning algorithms for demand forecasting, which is definitely a promising method worth more research in the future, due to the current capabilities and significant evolution that these algorithms have been characterized by recently.

From the literature discussed above, Table 1 was compiled, with the most commonly used explanatory variables in demand forecasting, as a summary. In the last line of the table, there is the key data from the present work. It is presented in the same shape of the literature reviewed, in order to allow an easier comparison. The data that was included in the final demand model is presented, organized in columns with different categories of variables. An in depth analysis and a description of the development of the predictive model will be presented in chapter 5.

Table 1: Summary of relevant documents in demand prediction literature

	Population variable	Economic variable	Price of transport	Frequency of transport	Duration of trip	Geographic variable	Importance of tourism
Calderon, J.D.J, [27]	Sum of inhabitants of O&D	Weighted average income per capita of O&D	Cheapest fare on the route	number of return weekly flights	N/A	Distance between airports	Ratio of hotel guests to inhabitants
Erjongmanee & Kongsamutr [36]	Population of O&D	GDP/capita of O&D	Average cost of ticket	N/A	Travel time between O&D	Distance between airports	Number of tourists of O&D
Carmona-Benítez et al. [33]	Economically active population	Consumer price index	N/A	Total number of flights in each airport	N/A	N/A	Hotel occupancy index
Devoto et al. [35]	Resident population in the market	N/A	N/A	N/A	N/A	N/A	Number of tourist beds, tourist arrivals
Demirsoy [31]	Population in the market	Average Income, oil prices	N/A	N/A	N/A	N/A	N/A
Adeniran & Adeniran [32]	N/A	Change in: currency value and GDP	N/A	N/A	N/A	N/A	N/A
Kluge & Paul [34]	Population in the market and degree of urbanization	GDP/capita	N/A	Number of air trips per capita	N/A	Variable defining whether the country is in an island	N/A
Wadud [29]	Population of O&D, separate variables	GDP/capita of O&D, separate variables	Ratio of cost by air to cost by road	N/A	Travel time ration between air and road travel	N/A	N/A
Present Work	Logarithm of Product of O/D Population	N/A	N/A	Frequency of Flights	Travel time between O&D	Distance between O&D	N/A

2.2.2. Fleet Optimization

Due to the importance of transportation in general, and particularly airline transport to the economic development of regions, there is significant literature published addressing the optimization of the usage of airline's resources, namely airplanes and crews.

Lohatepanont & Barnhart [37] and Sherali et al. [38] are two widely recognized publications that assess the problem of flight scheduling and fleet assignment with the sole purpose of maximizing profit for airlines. These papers obtained interesting improvements in their case studies, with several publications building on this objective, which should be expected, due to the financial interest associated with this type of optimization. One of these, published more recently, is Jamili [39] , which has the same objective, although exploring different methodologies to achieve it.

Also, Liu et al. [40] analyzes this problem through a different approach: by minimizing the impacts of the inevitable delays that are associated with airline operation, and imply significant expenses for the airlines. This is achieved through a combination of a traditional genetic algorithm with a multi-objective optimization method, addressing multiple objectives (such as turn-around times, flight connections, flight swaps) simultaneously, and then exploring the optimal solution. This is interesting because it would not be realistic to plan operations assuming everything will always go according to schedule, hence it is important for academic literature to also focus on schedule disruption, and how to minimize the associated effects.

The main reference for this dissertation came from Pita et al. ([18] and [19]), where a model was built that, instead of focusing on maximizing economic results for airlines, adds the objective of the maximization of the quality of the service provided to passengers. The models are applied to case studies in the PSO networks established in the Azores and in Norway, respectively. The second paper, builds on the first one, taking also into account the expenses and revenues of airport owners, associated with these routes. Both case studies obtained very interesting results, reducing costs in all the areas considered, and with impressive computational times required to reach the optimum solution. This is the reason why these papers are the main reference for the present research, with the purpose of adapting this model to a Greek PSO network.

Continuing the previously mentioned research, Antunes et al. [41] focused on analyzing in depth the network of the Azores operated by SATA, working closely with the airline. This allowed real data to be used as much as possible, reducing the amount of assumptions. With the objective of analyzing the maximum reduction of operating costs that SATA could have achieved by optimizing the network and changing its route structure. This was done while satisfying the same passenger demand as in 2012, with the current fleet, taking into account the implications of possible changes on the level of service offered. The paper proposed new shapes for the PSO imposed network, and quantified the improvements that could be obtained, with real data from a year in the past. The research concluded that the variable operating costs could be reduced significantly, which would save the government of the Azores a significant amount of funds in subsidies.

Iliopoulou et al. [42] was analyzed due to the similarities it has with the present research. This paper proposes a sea-plane network in the Greek islands, which would compete against the locally well-established ferry boat network. The objective is to minimize the travel cost, the size of the fleet and the unsatisfied demand between successive island ports, by proposing a new network, instead of optimizing an existent route, hence it is not straightforward to assess the improvements achieved. Also with the goal of proposing a completely new network is the case study developed by Dozic & Kalic [43] which, in three stages, performed a comprehensive analysis into all the steps required for designing the new route. This included defining the appropriate fleet mix, fleet size and aircraft selection, with positive results.

Ma et al. (2017) [44] addresses arguably one of the most discussed problems currently, which is the need for the reduction of carbon dioxide emissions. It develops an optimization model whose objective is to maximize the result of profit minus emissions, hence, the research aims at simultaneously maximizing profit and minimizing emissions associated with operating the flights. This model is applied to case studies from Asian airlines, with interesting results, namely that the optimal point obtained mathematically proved to be unreachable in real life. Besides this, the point achievable in reality that was closest to optimality had significant improvements over the current situation, and it was concluded that small reductions in profits lead to significant reductions in emissions.

Once again, the results above were all compiled into Table 2 , for a better comparison of the results.

Table 2: Summary of relevant documents in fleet optimization literature

	Objective	Case study	Time to find optimum solution	Order of magnitude of improvement	Solution method
Lohatepanont & Barnhart [37]	Maximize profit to the airline	Undisclosed major US airline	12 hours	5%	Branch and Bound
Liu et al. [40]	Minimize delays, flight swaps and connections	Reopening of Sungshan and Taichung airports	Not more detailed than "minutes"	Not disclosed	Multi-objective Genetic Algorithm
Sherali et al. [38]	Maximize net revenue	United Airlines	24 hours	10%	Benders Decomposition
Pita et al. [18]	Minimize operating costs for the airline and travel time	PSO network in the Azores	88 minutes	10%	Branch and Bound
Pita et al. [19]	Minimize operating costs for both the airline and airport, travel time	PSO network in Norway	20 hours	30%	Branch and Bound
Iliopoulou et al. [42]	Minimize travel cost, number of route, passengers not served	Prospective Seaplane network in Greece	45 minutes	N/A	Genetic algorithm coupled with a hybrid process
Dozic & Kalic [43]	Minimize fleet required, operating costs	Hypothetical airline based in Belgrade	Not disclosed	N/A	Fuzzy logic, heuristic, analytic approaches and multi-criteria decision making
Jamili [39]	Maximize profit to the airline	Not disclosed	1 hour	N/A	Hybrid with simulated Annealing and particle swarm optimization
Ma et al. [44]	Alternatively maximize profit or minimize emissions	Jetstar Asia and undisclosed Chinese airline	Not disclosed	8%	Compromise method to the Pareto Solution
Present work	Minimize operating costs for the airline and travel time	Two segments of Greek PSO network	24 hours	10%	Branch and Bound

In the last line of this table, the features of the present work are presented, in order to allow for a better comparison with the remaining literature, which was reviewed above. The model developed in this work will be analyzed and discussed in Chapter 3.

3. Optimization model

3.1. Integer Linear Programming Formulation

For the optimization model, a significant amount of input data must be specified. Then, as many constraints as possible must be defined, as well as the objective function. With these steps complete, the software program is able to solve the integer linear programming optimization problem.

3.1.1. Inputs

First, the following constants were defined:

1. NA, which defines the number of airports in the network;
2. NR, which defines the number of aircraft types available;
3. NWT, which defines the number of possible time periods waiting on the ground, by a passenger, for a connecting flight;
4. NT, which defines the number of time periods for the calculation;
5. NF, representing the number of possible flight routes, one between each O/D pair in each way.

Then, the associated sets were defined:

1. A, ranging from 1 to NA, has all the airports;
2. R, ranging from 1 to NR, has all the aircraft types;
3. WT, ranging from 1 to NWT, has all the possible waiting times;
4. T, ranging from 1 to NT, has all the time periods;
5. FL, ranging from 1 to NF, has all the possible flight routes.

With the sets defined, the inputs to the model were created, which are a function of these sets:

1. c_B: cost of time for being on board an aircraft for the passengers (euros/h);
2. c_W: cost of time for waiting on the ground for the passengers (euros/h);
3. z(r): number of aircraft of each type;
4. s(r): seat capacity of each aircraft type;
5. x_{min}(f): minimum number of flights in flight route f, as imposed by the PSO;
6. s_{min}(f): minimum number of seats available in flight route f, as imposed by the PSO;
7. t_F(f): travel time to complete flight route f;
8. $t_A(a_1,a_2)$: travel time between airports a_1 and a_2;
9. c_F(r): direct operating cost to perform a flight with aircraft type r per time period;
10. c_S(a,r): cost of having an aircraft of type r on the ground, in airport a, per time period;
11. q(f): demand for flight route f;
12. l(f): maximum load factor that the airline will sell, for flight route f;
13. d_F(f): specifies which is the airport of departure for flight route f;

14. $a_F(f)$: specifies which is the airport of arrival for flight route f;

One important remark is that inputs 7 and 8 represent the same travel time, but number 7 is in the form of a vector, and number 8 in the form of a matrix. This was necessary because the decision variables which will be presented in section 3.1.3, u_D and u_2 only have as inputs the route f, whereas u_1 is referenced to one route f and one airport a.

All these inputs will be read from a file named "inputs.dat", created while processing the data collected for the case study.

3.1.2. Pre-processed variables

In order to reduce the computation time, pre-processed variables were used in order to avoid having the software searching for the optimal solution in unreasonable solutions (e.g. searching in a connection itinerary arriving at the initial departure airport). The chosen pre-processed variables will be assigned the value of 1 for possible entries, and the value of 0 to impossible entries.

The chosen pre-processed variables were:

1. FF(f,t): equal to 1 if the flight route f, leaving at time period t arrives at destination until the last time period;

2. $d_1(f,a,t,wt)$: equal to 1 if flight itinerary which includes flight route f, and then continues to final destination a, represents a possible and reasonable itinerary, first departing at time t and waiting for time wt for a connection;

3. $d_2(f_1,f_2,t,wt_1,wt_2)$: equal to 1 if itinerary which includes 3 flights, being the first and last, respectively, f_1 and f_2, and the second flight the route that connects the arrival of f_1 with the departure of f_2 is plausible. This method reduces the amount of memory required to run the optimization, allowing for a more complex problem to be solved. Only the itineraries that are possible and make sense will have the value of 1 (e.g. an itinerary that in the end returns to the initial airport would never make sense for a passenger, hence will have the value of 0).

3.1.3. Decision Variables

The variables whose values will be optimized when running the model, the decision variables, are:

1. y(a,t,r): number of aircrafts of type r that are on the ground in airport a, from time t to (t+1);
2. x(f,t,r): number of aircrafts of type r that fly route f, departing at time t and arriving at time $[t+t_F(f)]$, this variable is defined as binary, in order to prevent the software considering the possibility of having several aircrafts flying the same route departing at the same time;
3. $u_D(f,t)$: number of passengers assigned to route f, taking off at t and landing at $[t+t_F(f)]$;

4. $u_1(f,a,t,wt)$: number of passengers assigned to the one stop itinerary which contains route f, and then continues to final destination a. Initial departure time is t, waiting time on the ground for connection is wt. Hence, the time of final arrival is given by $[t+t_F(f)+wt+t_A(a_F(f),a)]$.
5. $u_2(f_1,f_2,t,wt_1,wt_2)$: number of passengers assigned to the two stop itinerary which contains f_1 as the first flight and f_2 as the third flight, and has a flight joining the two airports as the second flight. Initial departure time is t and the waiting times on the ground are respectively wt_1 and wt_2. Hence, the time of final arrival is given by: $[t+t_F(f_1)+wt_1+t_A(a_F(f_1),d_F(f_2))+wt_2+t_F(f_2)]$;
6. $g_c(a,t,r)$: equal to 1 if aircraft r has been on the ground for more than 2 hours, starting at time t;

The graphical presentation of some of the first five variables is included in Figure 4, where the colors of u_D, u_1, u_2 are represented in boxes, demonstrating the meaning of each variable. It should be noted that this is not the most straightforward formulation, but with 56 routes, 8 airports and 33 time periods, initial simpler formulations would quickly have too many indexes for average computers to run out of memory (e.g. while this formulation for u_2 has 3 725 568 possible entries, the initial formulation had approximately 5.29×10^{12} entries, this means a reduction in the 10^6 order of magnitude).

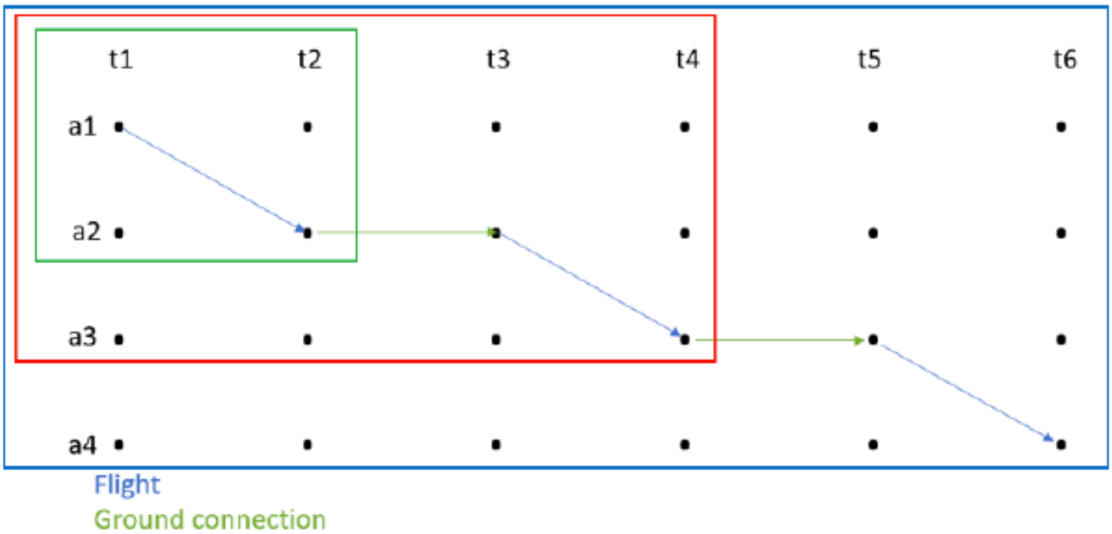

Figure 4: Possible itineraries for a passenger

These decision variables will be used in the applied constraints, the objective function, and the interpretation of their values will give the optimal solution to the problem.

3.1.4. Objective function

The objective function is what will be either maximized or minimized in the mathematical formulation. Hence, it is critical that the objective function is properly defined, and correctly reflects the physical reality of the problem. The objective function was defined as the sum of the following seven components (O_1 to O_7):

This first component (11) reflects the direct costs for the airline, resulting from the operation of the flights. It sums for all the flights performed, the product of the cost of each time period of flight in that aircraft, with the number of time periods the flight took and with the number of flights.

$$O_1 = \sum_{f \in FL} \sum_{t \in T} \sum_{r \in R} c_F(r).\, t_F(f).\, x(f,t,r) \tag{11}$$

The second component (12) accounts for the costs to the airline, of having an aircraft of type r, parked on the ground in airport a, at time t (when it exceeds 2 hours). This is achieved by summing, for all airports, aircraft types and time periods, the product of the respective parking cost with the number of aircrafts of that type parked in that airport, for more than 2 hours.

$$O_2 = \sum_{a \in A} \sum_{t \in T} \sum_{r \in R} c_S(a,r).\, g_c(a,t,r) \tag{12}$$

The third, fourth and fifth components [(13), (14), (15)] account for the social cost to the passengers, quantified by the cost of time for them, for direct, one stop and two stops itineraries, respectively. Hence, it is obtained by summing the product of all the time on board an aircraft with the number of passengers and the cost of time on board for passengers.

$$O_3 = \sum_{f \in FL} \sum_{t \in T} c_B.\, t_F(f).\, u_D(f,t) \tag{13}$$

$$O_4 = \sum_{f \in FL} \sum_{a \in A} \sum_{t \in T} \sum_{wt \in WT} c_B.\, [t_F(f) + t_A(a_F(f), a)].\, u_1(f,a,t,wt) \tag{14}$$

$$O_5 = \sum_{f_1 \in FL} \sum_{f_2 \in FL} \sum_{t \in T} \sum_{wt_1 \in WT} \sum_{wt_2 \in WT} cB.\, \big[t_F(f_1) + t_A\big(a_F(f_1), d_F(f_2)\big) + t_F(f_2)\big].\, u_2(f_1, f_2, t, wt_1, wt_2) \tag{15}$$

The sixth and seventh components [(16), (17)] account for the social cost to the passengers, of having to wait between two flights, on the ground, in an airport, respectively for one and two stop itineraries. Hence, it is obtained by summing the product of all the time on the ground in an airport, with the number of passengers and the cost of time on the ground for passengers.

$$O_6 = \sum_{f \in FL} \sum_{a \in A} \sum_{t \in T} \sum_{wt \in WT} c_W.\, (wt).\, u_1(f,a,t,wt) \tag{16}$$

$$O_7 = \sum_{f_1 \in FL} \sum_{f_2 \in FL} \sum_{t \in T} \sum_{wt_1 \in WT} \sum_{wt_2 \in WT} c_W . (wt_1 + wt_2) . u_2(f_1, f_2, t, wt_1, wt_2) \tag{17}$$

The objective function O is given by the sum of the above seven components, and the optimization problem will be defined by the minimization of O (min O).

3.1.5. Constraints

While implementing the model, several constraints were defined. Some constraints have the objective of defining the problem, others only have objective of reducing the required computation time, not being necessary to define the problem. The following constraints were defined:

The first constraint (18) ensures that the sum of aircrafts on the ground and in the air, at any time period, is equal to the available number of aircrafts of that type.

$$\sum_{a \in A} y(a,t,r) + \sum_{f \in FL} \sum_{\substack{t_1 \in T | \\ t_1 \leq t < (t_1 + t_F(f))}} x(f, t_1, r) = z(r), \ \ \forall\, t \in T, r \in R \tag{18}$$

The second constraint (19) imposes continuity in each node. It imposes that the sum of the number of aircrafts arriving into an airport and aircrafts already parked there, is equal to the sum of aircrafts departing from that airport and aircrafts that will stay parked there.

$$y(a, t-1, r) + \sum_{\substack{f \in FL | \\ a_F(f) = a \wedge t > t_F(f)}} x(f, t - t_F(f), r) = y(a,t,r) + \sum_{\substack{f \in FL | \\ d_F(f) = a}} x(f,t,r) \ \ , \forall a \in A, t \in T \setminus \{1\}, r \in R \tag{19}$$

Constraint (20) imposes that there are never more passengers assigned to a flight than the maximum allowed number of passengers to that flight. This is achieved by specifying that for all aircraft types, the sum of all passengers in direct or connecting flights is smaller or equal to the number of available seats. The number of available seats is calculated by summing for all aircraft types, the product of the number of aircrafts operating that route, with their capacity and with the maximum load factor. The use of the pre-processed variables d_1, d_2 and FF accelerates the computation of the solution, narrowing the search of the model.

$$\sum_{r\in R} l(f).s(r).x(f,t,r) \geq u_D(f,t) + \sum_{a\in A, wt\in WT} u_1(f,a,t,wt) + \sum_{\substack{f_1\in FL,\ a\in A, t_1\in T,\ wt\in WT| \\ a_F(f_1)=d_F(f)\wedge a_F(f)=a\wedge(t_1+t_F(f_1)+wt)=t}} u_1(f_1,a,t_1,wt)$$

$$+ \sum_{f_1\in FL,\{wt_1;wt_2\}\in WT} u_2(f,f_1,t,wt_1,wt_2) + \sum_{\substack{\{f_1;f_2\}\in FL,\ t1\in T,\ \{wt_1;wt_2\}\in WT| \\ a_F(f_1)=d_F(f)\wedge d_F(f_2)=a_F(f)\wedge(t_1+t_F(f_1)+wt_1)=t}} u_2(f_1,f_2,t_1,wt_1,wt_2) \tag{20}$$

$$+ \sum_{\substack{f_1\in FL,\ t_1\in T,\ \{wt_1;wt_2\}\in WT| \\ (t_1+t_F(f_1)+wt_1+t_A(a_F(f_1),d_F(f))+wt_2)=t}} u_2(f_1,f,t_1,wt_1,wt_2), \quad \forall f\in FL, t\in T$$

Constraint (21) ensures that the demand is satisfied, i.e. that all the passengers that must travel from one airport to another, will either be assigned to a direct, a one-stop or a two-stop itinerary.

$$\sum_{t\in T} u_D(f,t) + \sum_{\substack{f_1\in FL, t\in T, wt\in WT, a\in A| \\ d_F(f)=f_1\ \wedge\ a_F(f)=a}} u_1(f_1,a,t,wt) + \sum_{\substack{\{f_1;f_2\}\in FL, t\in T, \{wt_1;wt_2\}\in WT| \\ d_F(f)=d_F(f_1)\wedge\ a_F(f)=a_F(f_2)}} u_2(f_1,f_2,t,wt_1,wt_2) = q(f), \quad \forall f\in FL \tag{21}$$

Constraints (22) and (23) impose that, respectively, the minimum number of flights and seats between any two airports is fulfilled.

$$\sum_{t\in T,\ r\in R} x(f,t,r) \geq x_{min}(f) \quad ,\forall f\in FL \tag{22}$$

$$\sum_{t\in T,\ r\in R} s(r).x(f,t,r) \geq s_{min}(f) \quad ,\forall f\in FL \tag{23}$$

Constraints (24), (25), (26), (27) and (28) impose that, respectively the number of aircrafts on the ground, in the air, and passengers carried in direct, one-stop and two-stops itineraries are all positive integers.

$$y(a,t,r)\in\mathbb{Z} \quad ,\forall\, a\in A, t\in T, r\in R \tag{24}$$

$$x(f,t,r)\in\mathbb{Z} \quad ,\forall\, f\in FL, t\in T, r\in R \tag{25}$$

$$u_D(f,t)\in\mathbb{Z} \quad ,\forall\, f\in FL, t\in T \tag{26}$$

$$u_1(f,a,t,wt)\in\mathbb{Z} \quad ,\forall\, f\in FL, a\in A, t\in T, wt\in WT \tag{27}$$

$$u_2(f_1,f_2,t,wt_1,wt_2)\in\mathbb{Z} \quad ,\forall\,\{f_1;f_2\}\in FL, t\in T, \{wt_1;wt_2\}\in WT \tag{28}$$

Constraints (29) and (30) impose that the fleet starts and ends the day at the hub. This constraint was imposed due to information received from airlines operating these routes.

$$\sum_{f \in FL | d_F(f)=8} x(f,1,r) + y(8,1,r) = z(r) \quad , \forall\, r \in R \tag{29}$$

$$y(8,33,r) = z(r) \quad , \forall\, r \in R \tag{30}$$

Constraints (31) and (32) allow the model to only consider aircraft ground fees if an aircraft stays on the ground for more than 2 hours.

$$g_C(a,t,r) \in \{0,1\} \quad , \forall\, a \in A, t \in T, r \in R \tag{31}$$

$$\begin{gathered} g_C(a,t,r) \geq y(a,t,r) + y(a,t+1,r) + y(a,t+2,r) + y(a,t+3,r) - 3.5 \ , \\ \forall\, \mathrm{a} \in A, r \in R, t \in T \setminus \{31,32,33\} \end{gathered} \tag{32}$$

Constraint (33) imposes that there are not two different flights operating on the same route, with an interval smaller than 3 hours. This had to be imposed because one solution fulfilled all the frequencies imposed by the PSO with very small intervals, which is unreal.

$$\begin{gathered} \sum_{r \in R} [x(f,t,r) + x(f,t+1,r) + x(f,t+2,r) + x(f,t+3,r) + x(f,t+4,r) + x(f,t+5,r)] \leq 1, \\ \forall\, f \in FL, t \in T \setminus \{29,30,31,32,33\} \end{gathered} \tag{33}$$

Besides the above-mentioned constraints, which are necessary for the correct specification of the problem, other “virtual” constraints were added. The goal was to reduce the computation time to reasonable values and these constraints were added based on the concept of “helping” the model in narrowing down the range of possible solutions only to the reasonable ones. This removes from the scope of analysis of the software program unreasonable solutions, such as placing passengers in itineraries which end the day in the same airport as the departure.

This specification of additional “virtual constraints” must be carried out carefully, under the risk of removing the actual optimal solution from the range of possible solutions to be analyzed by the model.

Some examples of these constraints which were attempted, some with and some without success are:

1. Whenever one aircraft is departing the “hub” airport, all the fleet is departing the “hub” airport at that time. This potentializes the hub effect, and increases the possibility of connections in the hub, requiring less flights overall;
2. Imposing that in any moment in time there is a maximum of one aircraft operating in each route;
3. Specifying a maximum of one flight for the whole time of the analysis, for all the routes that have no minimum amount of flights assigned by the PSO network, or have low demand;

4. Impose that connecting itineraries which imply a total flown distance longer than 150% of the direct distance between origin and final destination do not have passengers placed there.

As detailed above, there is one objective function(O), to be minimized. This function is comprised of the sum of 7 components, and there are 16 equations imposing the necessary constraints for the model to reach a solution, which follows all the requirements for the problem.

In order to verify the correct solution by the model, an illustrative example was developed, which demonstrates that the model is correctly solving the given problem. This example will be presented next.

3.2. Illustrative example

3.2.1. Problem Specification

The objective of the example is to optimize the routes for a network comprised of 4 airports (1, 2, 3 and 4), during 10 time periods (each time period represents one hour, going from 10 am to 7pm). A specified demand of passengers (to be detailed below) must be fulfilled, and there are two types of aircraft available.

1. type A which can carry 60 passengers on board, has an operating (in flight) cost of 2000 €/h, a ground cost of 100 €/h and there is 1 aircraft of this type available;
2. type B, which carries 120 passengers on board, has an operating cost of 3000 €/h, a ground cost of 100 €/h and there is 1 aircraft of this type available.

It was imposed that no passenger will have to wait for more than 3 hours on the ground for a connection, and the demand to be fulfilled is presented in Table 3.

Table 3: demand to be fulfilled in the illustrative example

Demand					
		Arrival			
		Airport 1	Airport 2	Airport 3	Airport 4
Departure	Airport 1		5	10	20
	Airport 2	30		20	25
	Airport 3	120	51		70
	Airport 4	15	10	20	

Since this model is applied to public service obligation routes, which usually impose a minimum amount of flights and/or seats between certain airports, this is also implemented in the model. For this example, the following minimum number of flights and seats were imposed:

- 1 flight per day (round trip) between airports: 1-2, 1-4, 2-3;
- 60 seats from 1 to 2;
- 50 seats from 2 to 1;
- 120 seats from 1 to 4 and from 4 to 1;
- 100 seats from 2 to 3 and from 3 to 2;

The travel time between different airports is obviously variable and this is considered. For this example, the following travel times were defined:

- 1 hour between: 1-2, 1-4, 2-4, 3-4 (for each direction);
- 2 hours between: 1-3, 2-3 (for each direction);

Another feature of the model is to impose maximum allowed load factors (which is defined by the number of occupied seats, divided by the number of installed seats) for each route, which is something that airlines or even the PSO may impose. For this example, the following maximum load factors were imposed, as presented in Table 4.

Table 4: maximum load factor for the illustrative example

Maximum load factor (%)					
		Arrival			
		Airport 1	Airport 2	Airport 3	Airport 4
Departure	Airport 1		90	95	98
	Airport 2	100		98	99
	Airport 3	100	98		95
	Airport 4	100	99	100	

3.2.2. Results

The model solved the problem, giving the optimal solution in less than 3 seconds. This demonstrates how quickly the model converges to the optimal solution, even though the problem is of small scale and simple. As a comparison, the initial (more straightforward) formulation, which was already mentioned when defining the decision variables, took 6 minutes to reach the optimal solution for the same problem. Hence, a significant reduction in computational time was achieved between the previous and the final formulations. The solution is presented graphically below, in Figure 5, where each row represents an airport, and each column one hour of the day, from 10:00 to 19:00. In Figure 6, the detailed solution is presented.

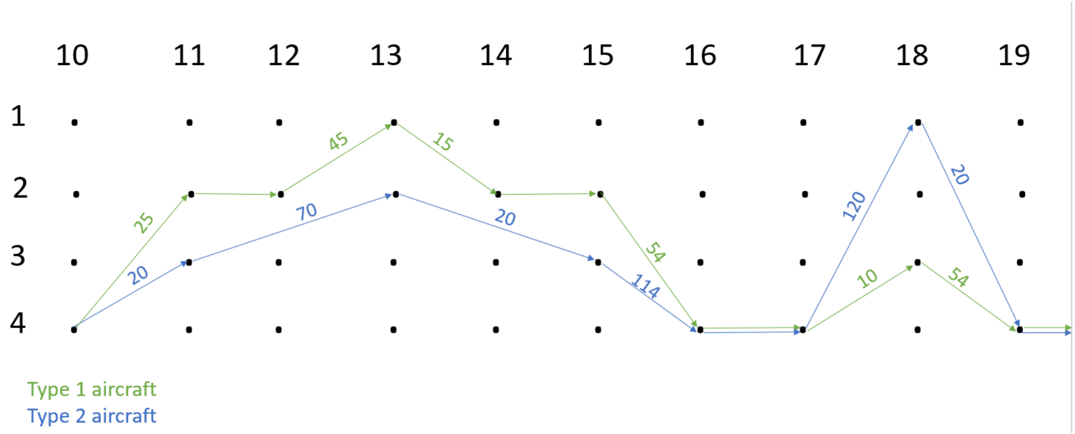

Figure 5: Graphical results from the illustrative example

```
The minimum cost is : 41135·
The flight operating costs are: 36000·
The aircraft ground costs are: 0·
The onboard time costs are: 3300·
The ground connection time costs are: 1835·
there was one flight performed departing from 4 at time 1 arriving at 2 at time 2, the type was 1 carrying 25 passengers aboard
there was one flight performed departing from 4 at time 1 arriving at 3 at time 2, the type was 2 carrying 20 passengers aboard
there was one flight performed departing from 3 at time 2 arriving at 2 at time 4, the type was 2 carrying 70 passengers aboard
there was one flight performed departing from 2 at time 3 arriving at 1 at time 4, the type was 1 carrying 45 passengers aboard
there was one flight performed departing from 1 at time 4 arriving at 2 at time 5, the type was 1 carrying 15 passengers aboard
there was one flight performed departing from 2 at time 4 arriving at 3 at time 6, the type was 2 carrying 20 passengers aboard
there was one flight performed departing from 2 at time 6 arriving at 4 at time 7, the type was 1 carrying 54 passengers aboard
there was one flight performed departing from 3 at time 6 arriving at 4 at time 7, the type was 2 carrying 114 passengers aboard
there was one flight performed departing from 4 at time 8 arriving at 1 at time 9, the type was 2 carrying 120 passengers aboard
there was one flight performed departing from 4 at time 8 arriving at 3 at time 9, the type was 1 carrying 10 passengers aboard
there was one flight performed departing from 1 at time 9 arriving at 4 at time 10, the type was 2 carrying 20 passengers aboard
there was one flight performed departing from 3 at time 9 arriving at 4 at time 10, the type was 1 carrying 57 passengers aboard
there were 10 passengers transported, leaving from airport 4 at time 1 and arriving at airport 2 at time 2
there were 20 passengers transported, leaving from airport 4 at time 1 and arriving at airport 3 at time 2
there were 51 passengers transported, leaving from airport 3 at time 2 and arriving at airport 2 at time 4
there were 30 passengers transported, leaving from airport 2 at time 3 and arriving at airport 1 at time 4
there were 5 passengers transported, leaving from airport 1 at time 4 and arriving at airport 2 at time 5
there were 20 passengers transported, leaving from airport 2 at time 4 and arriving at airport 3 at time 6
there were 25 passengers transported, leaving from airport 2 at time 6 and arriving at airport 4 at time 7
there were 13 passengers transported, leaving from airport 3 at time 6 and arriving at airport 4 at time 7
there were 20 passengers transported, leaving from airport 1 at time 9 and arriving at airport 4 at time 10
there were 57 passengers transported, leaving from airport 3 at time 9 and arriving at airport 4 at time 10
there were 1 aircraft(s) on the ground at airport 2 at time 2 and the type was 1
there were 1 aircraft(s) on the ground at airport 2 at time 5 and the type was 1
there were 1 aircraft(s) on the ground at airport 4 at time 7 and the type was 1
there were 1 aircraft(s) on the ground at airport 4 at time 7 and the type was 2
there were 1 aircraft(s) on the ground at airport 4 at time 10 and the type was 1
there were 1 aircraft(s) on the ground at airport 4 at time 10 and the type was 2
there were 15 passengers transported, leaving from airport 4 at time 1 with connection in airport 2 arriving there at time 2 and
leaving at time 3 to airport 1, landing at time 4

there were 101 passengers transported, leaving from airport 3 at time 6 with connection in airport 4 arriving there at time 7 and
leaving at time 8 to airport 1, landing at time 9

there were 19 passengers transported, leaving from airport 3 at time 2 with connection in airports 2 arriving at time 4 and
departing at time 6 to connect at airport 4 landing there at time 7 and departing at time 8 arriving at airport 1 at time 9

there were 10 passengers transported, leaving from airport 1 at time 4 with connection in airports 2 arriving at time 5 and
departing at time 6 to connect at airport 4 landing there at time 7 and departing at time 8 arriving at airport 3 at time 9
```

Figure 6: Detailed results from the illustrative example

It can be seen from the results presented in Figure 6 that the demand specified in Table 3 was correctly fulfilled. These results are obtained from a text file created by the optimization algorithm, detailing the solution.

As it was demonstrated, the constraints related with the demand were all followed, including the maximum time on the ground waiting for a connection flight (only once equal to 3 hours, and never greater).

Now, regarding the constraints related to the flights:

- It was required to have round trips between 1-2, 1-4, 2-3 which is verified;
- It was required to have 60 seats from 1 to 2, and 50 seats from 2 to 1. This round-trip flight is performed by a type 1 aircraft, so this constraint is verified;
- 120 seats from 1 to 4 and from 4 to 1. This round-trip flight is performed by a type 2 aircraft, so this constraint is verified;
- 100 seats from 2 to 3 and from 3 to 2. This round-trip flight is performed by a type 2 aircraft, so this constraint is verified.

The last verification that must be performed, is that the maximum load factors are never exceeded. In Figure 7, for each flight the number of passengers on board is written, followed by, in brackets, the maximum number of passengers allowed. As an example, a flight departing from airport 4 at 10:00 and arriving to airport 2 at 11:00 carries 25 passengers with a maximum of 60. This number was obtained by multiplying the maximum load factor for that route, with the capacity of the type of aircraft which operated the flight.

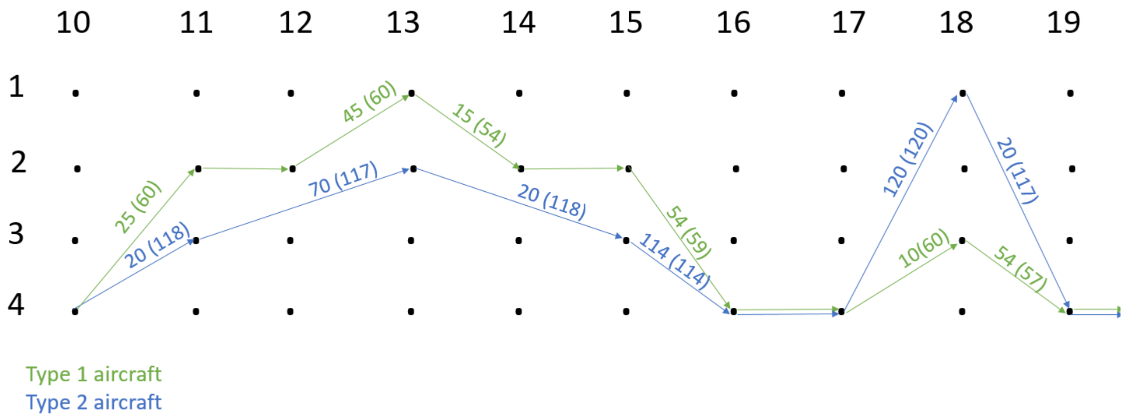

Figure 7: Demonstration of the capability to not exceed the maximum load factor

It can be verified that there were 29 passengers placed in flights with 2 ground connections because there was no availability left to allow more direct itineraries.

4. Two Greek Case studies

This dissertation applies the optimization model defined in the previous chapter to two case studies located in Greece, within the Greek PSO network. Hence, for each case study, one “based” in Rhodes airport and another “based” in Thessaloniki airport, the airports which had PSO routes imposed connecting them to the “base” airports were joined into two networks. The objective was to improve connectivity within the network, not resorting to the biggest hub in the country, Athens airport, for connecting flights. This objective was set due to the geographical proximity of the islands within each network, where a connection in Athens implies a long deviation.

As it was explained in the first chapter, the European PSO scheme has transparency as one of its core values. Hence, the information regarding each PSO network is made publicly available in the European Commission’s website [10]. With data extracted from that source, tables 5 and 6 below were compiled, with key information from the PSO networks which are the base for the case studies. An explanation of the information in the table will be given below.

Table 5: Key data regarding the PSO scheme [10]

Airport	Airport	No. of weekly return frequencies	PSO passengers in 2017	Load Factor (pax/minimum seats)	Annual economic compensation (€)	Type of aircraft
Rhodes	Karpathos-Kasos	3/4/6	18 741	141.98%	795 000	ATR-42
Rhodes	Kastelorizo	2/2/3	7 023	79.81%	919 199	Dash 8-100
Rhodes	Kos-Kalymnos-Leros-Astypalaia	3/4/6	3 415	56.45%	1 089 000	ATR-42
Thessaloniki	Kerkyra	2	15 547	166.10%	99 000	ATR-42
Thessaloniki	Limnos-Ikaria	3/4/6	14 646	110.95%	528 000	ATR-72
Thessaloniki	Samos	3/4/6	23 581	89.32%	N/A	Dash8-100/400
Thessaloniki	Skyros	2	2 496	40.00%	250 000	ATR-42
Thessaloniki	Chios	3/4/6	31 331	118.68%	N/A	Dash8-100/400
Thessaloniki	Kalamata	3/4/6	12 810	58.23%	N/A	Dash8-100/400

In order to facilitate the comprehension of the data included in Table 5, the values in the first column will be explained. The first two columns detail the airports involved, and should be read as a sequence, i.e. it is imposed that a flight must depart from Rhodes, with final destination Kasos, but with an intermediate stop in Karpathos. The return flight (i.e. Kasos to Rhodes, with an intermediate stop in Kasos) is also imposed automatically.

The third column specified the minimum weekly frequencies imposed by the PSO. In the lines where there are 3 values (e.g. 3/4/6), each value represents the imposed frequency for one season of the year, as the Greek aviation sector divides the year in three seasons, according to the expected demand. The highest values correspond to the summer months, the lowest values to the winter months, with the remaining value corresponding to the mid-season.

The fourth, fifth, sixth and seventh columns are self-explanatory. One remark about the fifth column explains why there are percentages of load factor greater than 100%. This is because the value is calculated as the number of actual passengers, divided by the minimum amount of seats imposed by the PSO, while typically load factor represents occupied seats divided by total available seats.

Table 6: Key data regarding the Greek PSO scheme (continuation) [10]

Airport	Airport	Annual seats required by the PSO	Open or Restricted PSO (O/R)	Number of bids in the tender process	Airlines Operating (names)	PSO in force from
Rhodes	Karpathos-Kasos	13200	R	2	Sky Express	1-Oct-2016
Rhodes	Kastelorizo	8800	R	1	Olympic Air	1-Oct-2016
Rhodes	Kos-Kalymnos-Leros-Astypalaia	6050	R	2	Sky Express	1-Jun-2018
Thessaloniki	Kerkyra	9360	R	3	Sky Express	12-Apr-2018
Thessaloniki	Limnos-Ikaria	13200	R	1	Astra Airlines	1-Oct-2016
Thessaloniki	Samos	26400	O	N/A	Astra Airlines, Olympic Air, Sky Express	1-Oct-2016
Thessaloniki	Skyros	6240	R	1	Sky Express	1-Apr-2017
Thessaloniki	Chios	26400	O	N/A	Astra Airlines, Olympic Air, Sky Express	1-Oct-2016
Thessaloniki	Kalamata	22000	R	1	Olympic Air	1-Oct-2016

In Table 6, the first two columns represent the same information as in Table 5, in order to understand to which network the information refers to. The third column specifies the minimum number of seats that must be made available annually to the public by the airline, as imposed by the PSO. The fourth column specifies if the route is part of an open or restricted PSO, which defines whether other airlines can offer competitive air services to the subsidized route. The fifth column details how many bids were submitted in the tender process, when imposing that specific route. In the sixth column, the name of the airline(s) which were granted the operation in that route is shown, and it should be noted that routes specified as restricted (R in the fourth column), only have one airline operating, as was explained in section 1.1.2. The last column specifies the date when that PSO network was imposed.

With the PSO impositions described, the networks will be presented. Each network is comprised of 8 airports, including the "hub" airport, with 56 possible routes. These networks were chosen because, although they have the same number of airports and are located relatively close to each other, they are different in terms of the number of aircrafts employed, passengers transported, and frequencies imposed by the PSO, which will allow for a more comprehensive analysis of the Greek market.

The goal of these case studies is to reduce the total costs of the networks to the lowest possible values. The total cost to be considered is the sum of the following four components:

1. Aircraft direct operating costs – this represents the direct cost for an airline to operate a flight, and includes the fuel, the crew, the airport and airspace fees, the maintenance, etc. Usually in the airline industry this cost is calculated per block hour (block time in aviation refers to the time the engines are operating in a flight), and this value will be used. Based on Eurocontrol's Standard inputs for cost benefit analysis document [45] the cost per block hour was estimated to be:
    a. 1502 €/h for the Bombardier dash8-q100;
    b. 1502 €/h for the ATR42;
    c. 2376 €/h for the Bombardier dash8-q400.
2. Aircraft ground costs – this represents the cost for the airline to have the aircraft on the ground and is considered to be the parking fees in the airport. These were estimated to be 10% of the aircraft direct operating costs [18]. It is common in the airline industry for airlines not to pay parking fees if the aircraft is on the ground for at least less than 2 hours [25];
3. Passenger on board time cost – this represents the cost of the time spent on board, for a passenger. It was estimated to be 10€/h for the passengers in these networks, of which business travelers account for a small percentage of the overall demand [45];
4. Passenger ground connection time cost – this represents the cost of the time spent in an airport waiting for a connection, for a passenger. It was estimated to be 10€/h for this network, assuming it is mostly comprised of tourist passengers. This is justified by the fact that for tourists, there is no significant difference between time spent on board or on the ground.

Having detailed the values, each network will be discussed separately. Section 4.1 specifies Rhodes network and section 4.2 specifies Thessaloniki network.

4.1. Rhodes Network

The first case study can be considered the most simple and Rhodes airport is considered its “hub”. It can be considered the most simple due to the smaller number of frequencies imposed, the smaller number of aircraft operating in the network and the overall smaller costs involved, when compared to the second case study.

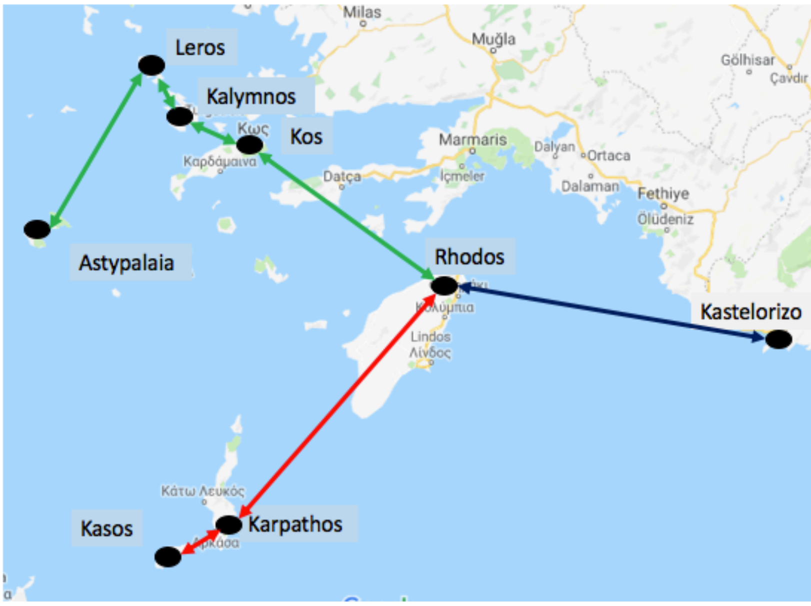

Figure 8: Graphical representation of the current Rhodes network.

In this network there are 7 routes imposed by the PSO, as represented in Figure 8. All of these routes are operated in both directions (hence 14 routes in total, if outbound and inbound legs are considered as different routes). The fleet that operates this network was considered to be composed by one Bombardier Dash 8 Q100 aircraft and two ATR 42 aircrafts. This results from an extensive analysis of the aircrafts operating in these routes, through flight tracking websites, leading to the conclusion that this was the most accurate representation of the real situation. The costs associated with this network are detailed below.

4.1.1. Aircraft Operating Costs

Regarding the aircraft operating costs, using real data provided by the HCAA from last summer, the number of movements and aircraft type associated with each O/D pair was noted, as represented in Table 7.

Table 7: Number of flights per O/D pair in the Rhodes network

Rhodes flights									
		Arrival							
		Astypalaia	Kalymnos	Karpathos	Kasos	Kastelorizo	Kos	Leros	Rhodes
Departure	Astypalaia		2	0	0	0	1	1	1
	Kalymnos	1		0	0	0	1	1	2
	Karpathos	0	0		4	0	0	0	4
	Kasos	0	0	4		0	0	0	3
	Kastelorizo	0	0	0	0		0	0	3
	Kos	1	1	0	0	0		0	1
	Leros	2	1	0	0	0	0		2
	Rhodes	0	2	3	3	3	1	2	

Using this data multiplied by the duration of each flight, the total aircraft operating cost for this network was estimated to be 53 771€, for a total of 50 flights, resulting in an average cost of 1 075€ per flight.

4.1.2. Aircraft Ground Costs

Regarding the aircraft ground costs, according to the information collected from the Greek airlines, airplanes are scheduled to stay for less than 2 hours on the ground. Since parking fees are only charged when an aircraft stays on the ground for more than 2 hours, it was considered that for the existing network, aircraft ground costs are zero.

4.1.3. Passenger Time Costs

Regarding the Passenger time costs, the total travel time for each O/D pair was compiled, and is presented in Table 8. These values were compiled through an extensive search from online travel websites, and for each O/D pair, the travel times of at least one full week were verified, and the shortest value was considered. This was done in order to allow for a fair comparison, due to the fact that there are days of the week which allow for better connections than others. Some routes have direct flights, whereas others have long connections, explaining the broad range of values for the travel time.

With this information, for each O/D pair the passenger time costs were calculated using the following expression:

$$PTC = Ct \times TT \times PN \tag{34}$$

Where Ct is the cost of time for the passengers, TT the travel time and PN the number of passengers in that route. The total passenger time costs were then calculated by summing all the passenger time costs for each O/D pair associated with that network. The total passenger time costs were estimated to be 6 470€ for a total of 375 passengers, resulting in an average travel time of 1 hour and 44 minutes per passenger.

Table 8: Travel time per O/D pair for the Rhodes network

Rhodes Travel time (hours)									
		Arrival							
		Astypalaia	Kalymnos	Karpathos	Kasos	Kastelorizo	Kos	Leros	Rhodes
Departure	Astypalaia		1.2	4.0	4.8	20.0	2.0	0.5	3.0
	Kalymnos	1.2		2.5	3.3	12.3	0.5	0.5	1.5
	Karpathos	5.5	7.0		0.3	9.8	7.8	5.3	0.6
	Kasos	14.0	13.5	0.3		14.3	8.3	17.5	1.3
	Kastelorizo	9.0	4.8	7.5	9.0		4.0	5.8	0.6
	Kos	1.9	0.5	1.6	2.3	8.5		1.2	0.5
	Leros	0.5	0.5	3.3	4.0	14.6	1.2		2.2
	Rhodes	2.8	1.3	0.6	1.3	0.6	0.5	2.0	

4.1.4. Total Cost

Summing all these components, the total cost for the Rhodes network was estimated to be 60 241€, for the period in analysis.

4.2. Thessaloniki Network

This network is also comprised of 7 routes, flown in both directions (hence 14 routes if considering outbound and inbound as 2 different routes), as presented in Figure 9. All of them are imposed by the PSO, fulfilled by 2 dash 8 Q100, 2 dash 8 Q400 and 1 ATR 42 aircrafts. The composition of the fleet in this network was obtained through the same process as in the previous network. The larger fleet size and geographical distances illustrate the larger dimension of this network, which contributes to the larger costs that will be verified next.

Figure 9: Graphical representation of the Thessaloniki network

The costs for this network were once again calculated and are presented below.

4.2.1. Aircraft operating costs

The same method was used to calculate the aircraft operating cost of this network, and the number of movements per O/D pair are presented in Table 9.

Table 9: Number of flights per O/D pair in the Thessaloniki network

Thessaloniki Flights									
		Arrival							
		Chios	Ikaria	Kalamata	Kerkyra	Limnos	Samos	Skyros	Thessaloniki
Departure	Chios		0	0	0	2	0	0	5
	Ikaria	0		0	0	3	0	0	3
	Kalamata	0	0		1	0	0	0	4
	Kerkyra	0	0	1		0	0	0	3
	Limnos	2	3	0	0		0	0	3
	Samos	1	0	0	0	1		0	5
	Skyros	0	0	0	0	0	0		1
	Thessaloniki	5	3	3	3	3	5	2	

Using this data multiplied by the duration of each flight, the total aircraft operating costs were estimated to be 134 424€, for a total of 62 flights, resulting in an average cost of 2 168€ per flight (which is more than twice the average operating cost for the Rhodes network).

4.2.2. Aircraft ground costs

For the same reasons as explained for the Rhodes network, the aircraft ground costs for the Thessaloniki network were assumed to be 0€.

4.2.3. Passenger time Costs

Regarding the Passenger time costs, using the same procedure as for the previous network, the total travel time for each O/D pair was compiled, and is presented in Table 10.

Table 10: Travel time per O/D pair for the Thessaloniki network

Thessaloniki Travel time (hours)									
		Arrival							
		Chios	Ikaria	Kalamata	Kerkyra	Limnos	Samos	Skyros	Thessaloniki
Departure	Chios		4.0	3.2	4.0	3.8	0..5	4	1.5
	Ikaria	4.5		16.5	3.3	0.8	3.0	13.3	1.5
	Kalamata	4.5	15.2		8.0	10.8	14.0	12.0	1.5
	Kerkyra	3.0	3.8	2.8		4.5	2.5	4.8	1.0
	Limnos	1.8	0.8	6.0	2.5		2.8	5.3	1.0
	Samos	0.6	3.2	60	3.6	2.8		4.2	1.3
	Skyros	3.5	4.3	4.8	2.8	3.0	5.0		0.8
	Thessaloniki	1.6	1.2	1.5	1.3	1.5	1.0	3.0	

Using equation (34), the total passenger time costs for the Thessaloniki network were estimated to be 27 637€ for a total of 1357 passengers (three times more passengers than in the Rhodes network), resulting in an average travel time of 2 hours and 1 minute per passenger.

4.2.4. Total Cost

Summing all the components above, the total cost for the Thessaloniki network was estimated to be 162 061€, for the period in analysis.

5. Predictive Model

This chapter will deal with the development of the predictive model, which will estimate values of demand for the routes in the optimization model that do not have available real world data.

5.1. Context

As mentioned previously, airlines operate under very tight operational constraints, with significant pressure over their margins. This way, it is not uncommon to see airlines that were profitable in one year, to have significant losses in the following year, only due to relatively small changes in the economic and operational environment. Hence, airlines must focus on optimizing their operational conditions, which will allow them to maximize their chances of success. In order to achieve this, one of the key decisions is the network that the airline will operate, and how to use the fleet in this network.

Therefore, the most important input to the optimization of an airline's network is predicting the demand for the associated routes. This prediction has to be performed for the current network and for O/D pairs which are candidates to future routes (which, as a process, has even more uncertainty associated). Due to its relevance, this task has attracted significant attention from academic research, as mentioned in chapter 2. This prediction task is very complex, and always involves significant uncertainty in the estimated demand (as analyzed in the book by Neufville and Odoni [25], which dedicated a whole chapter to this subject), a detail which must never be disregarded by the user of the estimation. There are several guidelines generally accepted which detail how to perform a demand prediction and quantify the associated error. The optimization model will need to have as an input the demand for the network in analysis, and because there is no demand data available covering all the network, it must be predicted following the above mentioned guidelines. The network is comprised of 16 airports, hence, there are 240 (16x15) O/D pairs, whose demand must be predicted.

5.2. Definition of explanatory variables

This demand will be estimated through multiple variable linear regression analysis, since this is a method commonly accepted in the literature concerning this area. With the objective of using published literature as a guideline, a literature review was carried out through published papers which analyzed the problem of demand prediction, in order to choose the most suitable explanatory variables to the case study. This literature review was already presented and discussed in section 2.2, but a summary of the most common explanatory variables found in these publications was defined, and is presented below:

1. **GDP** (either summing or multiplying both origin and destination, either total or per capita);
2. **Population** (either summing or multiplying both origin and destination);
3. **Importance of tourism** (either by number of tourist arrivals, hotel beds or per capita beds);
4. **Cost of ticket** (either absolute, or compared to its competition (e.g. rail, car, boat…));
5. **Travel time;**
6. **Distance between airports**.

Since this case study has some particularities, other variables that could describe these particularities were considered, and later their significance was assessed through the multiple variable linear regression analysis, such as:

1. Existence of significant cruise ship terminals in the islands, since it is expected that the embarking and disembarking of cruise ship passengers will increase air travel demand in the island. Hence, it is believed that if an island has a cruise ship terminal with significant activity, demand for airline tickets will increase;
2. Competition of the ferry boats, since this is a well-established and very popular mean of transportation within the Greek archipelago. The higher the quality of the service provided by the ferry boat (either through fast travel times, or low ticket prices), the lower the demand for airplane seats is expected to be.
3. Effect of population ageing. The hypothesis that areas with a higher share of retired population would have proportionally less travelling will be tested. This hypothesis comes from the fact that business activity promotes travelling. Hence, and since the average age of the population of some islands is above the Greek average, this was considered, through the percentage of residents above 60 years old, for each island. It is believed that the higher this percentage, the lower the demand for airline flights will be;

These additional variables and those mentioned previously taken from the reviewed papers will be analyzed, in order to determine which ones have the highest correlation with demand, for the network of the case study.

5.3. Collection of data

The first step in the multiple variable linear regression analysis was the data collection, namely the values of the above explanatory variables, for each of the 240 O/D pairs (for the predictive model, both networks were joined into one). This data was collected through several sources, and refers to the month of August 2018, namely:

1. For the sociodemographic variables: Population, GDP per capita, and population ageing, values were taken from official statistics sources such as Eurostat or the Hellenic Statistic Authority;
2. For the variables describing the transport market: existence of a ferry between origin and destination, cost of airplane ticket, cost of ferry ticket, existence of direct flight, airplane travel time (including connections if applicable), direct distance between airports and minimum number of weekly flights, as established by the PSO, values were taken from the official website of the European commission (for the PSO specific values), and from online travel websites (for the ticket prices, travel times and the existence of direct flights or ferries);
3. For the economic variables, related to tourism: sum of number of hotel beds in origin and destination and existence of cruise ship terminals, information was retrieved from the Hellenic

Statistic Authority, and from official tourism websites, developed by the national and local governments;

4. As many demand values as possible were obtained from the HCAA (passengers carried in scheduled direct flights between origin and destination) and from the connecting tickets emitted by Aegean airlines, which were then the base for predicting the missing values. In total, values for 54 direct routes, and 12 connecting routes were obtained. The creation of two demand models was considered, one with the data from direct routes only and another with the data from connecting itineraries, but it was decided that the number of routes for which the data was available was not enough for two separate models. Hence, the decision was to create a single model sustained by the 66 routes with available demand values.

Table 11: Example of collected sociodemographic and economic data

		SOCIODEMOGRAPHIC				**Economy - tourism**		
Origin	Destination	Population product ($x10^{-7}$)	GDP/capita product ($x10^{-7}$)	% over 60 (O)	% over 60 (D)	Product of hotel beds/capita	Cruise availability (O)	Cruise availability (D)
Astypalaia	Chios	7.027	24.723	0.225	0.270	0.019	0	1
Astypalaia	Ikaria	1.124	25.692	0.225	0.345	0.044	0	0
Astypalaia	Kalamata	57.680	28.060	0.225	0.309	0.005	0	1
Astypalaia	Kalymnos	2.158	32.964	0.225	0.232	0.038	0	0
Astypalaia	Karpathos	0.831	32.964	0.225	0.277	0.346	0	0
Astypalaia	Kasos	0.145	32.964	0.225	0.225	0.021	0	0
Astypalaia	Kastelorizo	0.066	32.964	0.225	0.225	0.074	0	0
Astypalaia	Kerkyra	13.616	31.024	0.225	0.281	0.158	0	1
Astypalaia	Kos	4.454	32.964	0.225	0.168	0.541	0	0
Astypalaia	Leros	1.056	32.964	0.225	0.225	0.053	0	0

Table 12: Example of collected transport market data

		TRANSPORT					
Origin	Destination	Sea connection	Cost Ticket Air (€)	Cost Ticket Boat (€)	Travel time (hours)	distance (NM)	Frequency Of Flights
Astypalaia	Chios	0	191	0	10.0	106	0
Astypalaia	Ikaria	0	168	0	16.5	66	0
Astypalaia	Kalamata	0	0	0	0.0	211	0
Astypalaia	Kalymnos	1	70	15	1.0	36	13
Astypalaia	Karpathos	0	180	0	16.0	79	0
Astypalaia	Kasos	0	0	0	0.0	74	0
Astypalaia	Kastelorizo	1	0	35	0.0	157	0
Astypalaia	Kerkyra	0	193	0	9.0	356	0
Astypalaia	Kos	1	98	17	9.0	37	13
Astypalaia	Leros	0	76	0	0.5	42	13

Once this information was collected (an extract is presented in tables 11 and 12), the regression was carried out using IBM's SPSS software package, through a Poisson regression. This type of regression was chosen due to its greater suitability to these types of data sets, with very different values of the dependent variable (demand), which depend on explanatory variables by a power different than one.

5.4. Choice of model

The regression went through several specification tests, in order to reach the most reasonable model possible, such as:

1. Checking for overdispersion of data, through the Lagrange Test, in order to validate either the Poisson regression or the negative binomial regression as the best option;
2. Verification of the statistical significance of the parameters, through the Wald test and p-values;
3. Analysis of the predictive capacity, through the Omnibus test;
4. Comparison between models with different specifications, in order to choose the most suitable one.

The initial approach to the regression was to use all the proposed explanatory variables, to analyze the result. Hence, it was assumed that the following explanatory variables (18 in total) would have a (positive or negative) impact on demand and this assumption was analyzed:

- Product of the Populations of the origin and destination markets, expected to have a positive impact on the demand, as its value increases;
- Population of the origin and destination markets, each one considered a separate variable, expected to have a positive impact on the demand, as its value increases;
- Product of the GDP/capita of the origin and destination markets, expected to have a positive impact on the demand, as its value increases;
- GDP/capita of the origin and destination markets, each one considered a separate variable, expected to have a positive impact on the demand, as its value increases;
- Fraction of the population older than 60 years, for origin and destination, each one considered a separate variable, expected to have a negative impact on demand, as its value increases;
- Dummy variable equal to 1 if the O/D pair is connected by ferry boat service, expected to have a negative impact on the demand, as its value increases;
- Cost of a ferry boat ticket for the O/D pair; if there is no ticket offered, will have the value of zero, expected to have a positive impact on the demand, as its value increases;
- Dummy variable equal to 1 if the O/D pair is served by a direct flight, with at least one weekly frequency, expected to have a positive impact on the demand, as its value increases;
- Cost of an airplane ticket, from the origin to the destination, even if this implies having connections. If there is no ticket offered (even with connections), will have the value of zero, expected to have a negative impact on the demand, as its value increases;
- Total travel time (in hours), expected to have a negative impact on the demand, as its value increases. Corresponds to the time required to reach the destination from the departing time of the first flight and includes the time waiting for ground connections, for non-direct flights;
- Straight line distance (in nautical miles), between origin and destination airports, expected to have a positive impact on the demand, as its value increases;
- Frequency of direct flights, for the period of analysis (one month). If there is no direct flight, the value will be zero, expected to have a positive impact on the demand, as its value increases;
- Product of the number of hotel beds/capita, from the origin and destination, expected to have a positive impact on the demand, as its value increases, due to the importance of tourism;
- Dummy variable equal to 1 if there is a cruise ship terminal in the location. One variable for origin and one for destination, expected to have a positive impact on the demand, as its value increases, due to the arrival and departure of, respectively, embarking and disembarking cruise ship passengers;
- A Dummy variable to be used as an offset variable, which would have the value of 1 for the biggest markets, and the value of 0 for the smaller markets.

With the data collected, the first step carried out was to run a negative binomial regression, in order to verify the result of the Lagrange test, that would indicate the adequacy of this model type. The Lagrange multiplier test gave a non-significant value, as seen in Table 13, which led to the decision that a Poisson regression would be a better option for this dataset.

Table 13: Result of the Lagrange multiplier test

	Chi-Square	df	Sig.
Ancillary Parameter	0.031	1	0.859

5.5. Regression analysis

Having taken the decision to use a Poisson regression, the next step was to attempt to use a fixed scale parameter method. Running this regression, the following results were obtained:

Table 14: Test of Model effects for the first regression

Source	Wald-Chi Square	Sig.
(Intercept)	2136.234	0.000
Sea connection	22.410	0.000
Direct Flight	5.685	0.017
Cruise availability (O)	427.836	0.000
Cruise availability (D)	303.122	0.000
Population Product	175.005	0.000
Travel Time	743.112	0.000
Distance	2037.116	0.000
Frequency of Flights	2425.649	0.000
Gdp/capita Product	1777.669	0.000
Cost Ticket Air	228.769	0.000
Product of beds/capita	524.395	0.000
% over 60 (O)	29.085	0.000
% over 60 (D)	22.087	0.000
Cost Ticket ferry Boat	5.010	0.025

Table 15: Goodness-of-fit values for the first regression.

	Value	df	Value/df
Deviance	5432.081	49	110.859
Scaled Deviance	5432.081	49	
Pearson Chi-Square	6920.854	49	141.242
Scaled Pearson Chi-Square	6920.854	49	
Log Likelihood	-2909.472		
AIC	5848.944		

Looking at the deviance value, which is significantly greater than 1, it was concluded that for this dataset, a fixed scale parameter method was not suitable, and the Pearson chi-square scale parameter method would be the acceptable option. Then, the next step was to evaluate which variables were significant, and would be included in the model, and which variables were not significant, and would be excluded from the model.

An extensive “trial and error” procedure was carried out, adding variables one by one, verifying its statistical significance and coefficient for predicting demand. It was also attempted to use different versions of the variables which were being excluded, in order to try and find other significant variables to include in the model. Hence, by using the logarithm of the product of the origin and destination population, this variable became significant and had a positive coefficient, which was expected, and agrees with published articles. This led to the replacement of the product of the population with the logarithm of the product of the population.

It was attempted, for the remaining variables which were not considered significant, to use the logarithm, exponential or dividing one by another, in various logic combinations, without success. This led to the decision that the model already had a reasonable number of explanatory variables, for the dataset.

The variables which were not significant, hence not included in the model were:

- Cost of the ferry boat ticket, believed not to be significant due to the fact that these prices are fixed, and very similar for almost all O/D pairs;
- Existence of ferry boat connection between origin and destination;
- Percentage of inhabitants over 60 years old, either at the origin or destination;
- Existence of a direct flight between origin and destination.
- GDP/capita, which is one of the most common variables used in this type of regression. It is believed that it was not significant in this case, because there is no publicly available data of GDP for each specific island. Hence, it was required to use the available data which exists for each NUT3 in Greece. This brings a problem which is the fact that, since several islands are on the same NUT3, the model considers the same value of GDP. The result is making this variable non-significant for this dataset, which lead to its exclusion from the regression.
- Hotel beds/capita;
- Cruise availability.

After this, the variables that were considered significant were:

- Travel time;
- Distance;
- Frequency of flights;
- Cost of air ticket;
- Product of the population in the origin and destination;

5.6. Final model

The final step was to gather key performance indicators on a small group of the best performing models, and compare the indicators of each model, in order to choose the final model to be used. This is presented in Table 16.

Table 16: KPI's of the different predictive model candidates

Model	**M1**		**M2**		**M3**		**M4**		**M5**	
offset included?	No		No		Yes		No		Yes	
	Beta	Std Dev.	Beta	Std Dev.	Beta	Std Dev.	Beta	Std Dev.	Beta	Std Dev.
Intercept	2.56***	0.63	2.21***	0.47	2.21***	0.45	3.57***	0.66	3.38***	0.51
Log Population Product	0.56***	0.12	0.30**	0.14	0.30**	0.10	0.47***	0.11	0.19*	0.09
Distance	0.01*	0.01	0.01***	0.01	0.01***	0.02	0.01**	0.01	0.01***	0.01
Frequency of flights	0.09***	0.02	0.10***	0.02	0.11***	0.02	0.07***	0.02	0.07***	0.02
Cost ticket air	-0.01*	0.01	----------	--------	----------	-----------	-0.01*	0.01	----------	-----------
Big market	----------	----------	0.10***	0.29	----------	-----------	----------	-----------	----------	-----------
Travel time	----------	----------	----------	-------	----------	-----------	-0.46*	0.21	-0.55**	0.12
Goodness of fit:										
AIC	8710.9		7414.4		7412.4		7749.1		6339.7	
log-likelihood	-4350.5		-3702.2		-3702.2		-3868.6		-3164.8	
Deviance	8314.1		7017.5		7017.5		7350.3		5942.8	
Pseudo R^2	0.687		0.734		0.734		0.722		0.772	
R^2 (Domencich and Macfaden (1975))	1		1		1		1		1	

Table 17: Key for the p-values of the explanatory variables

*: P value<=0.15
**: P value<0.10
***: P value<0.01

The Akaike's Information Criterion (AIC), the Log Likelihood and the Deviance were extracted directly from SPSS, while the Pseudo R^2 was calculated through:

$$\rho^2 = 1 - \frac{LL(\beta U)}{LL(\beta R)} \tag{35}$$

with $LL(\beta U)$ the log-likelihood of the unrestricted model, and $LL(\beta R)$ the log-likelihood of the restricted model. With the pseudo R^2 calculated, the R^2 was obtained through the empirical relation set by [46].

After carefully assessing the key parameter indicators of these 5 models, it was decided to calculate the demand for models 2 and 5, which were the models with the best indicators.

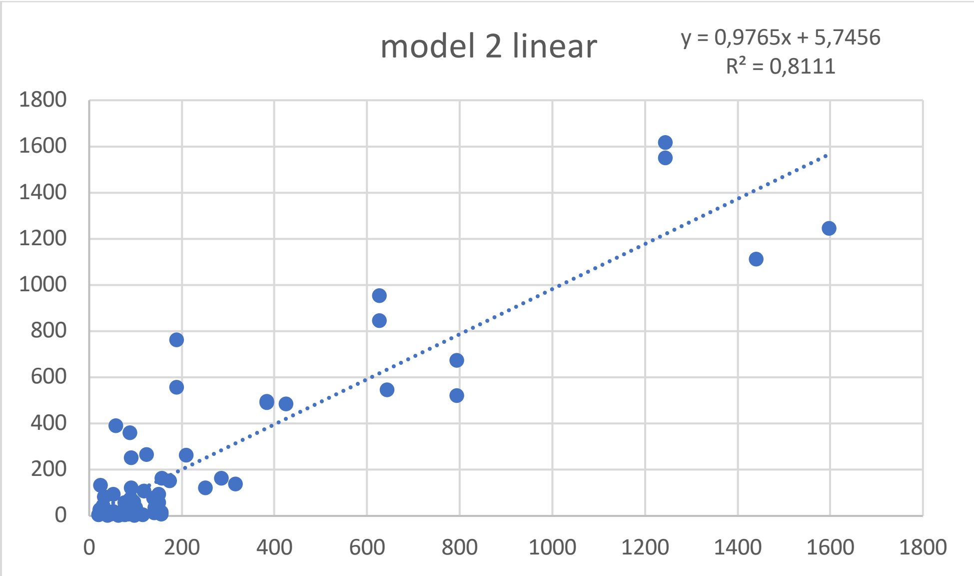

Figure 10: Comparison between expected and real values for model 2 on a linear scale

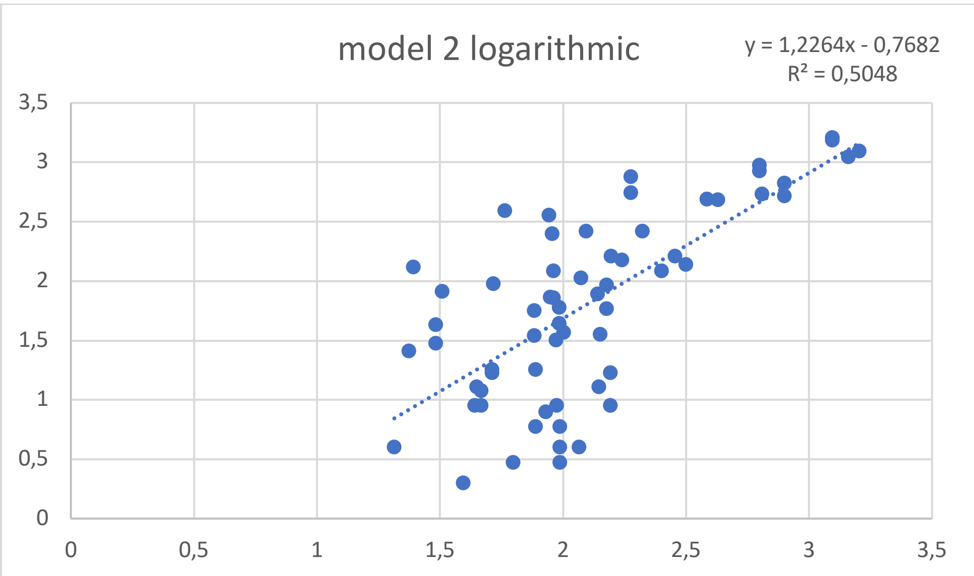

Figure 11: Comparison between expected and real values for model 2 on a logarithmic scale

After calculating this demand, the actual demand was compared with the predicted demand for the 66 routes, for which there is real world data available. This comparison was plotted into a chart, using the values in linear and logarithmic scales, which are presented in Figures 10 and 11 for model 2, and figures 12 and 13 for model 5. For the remaining models which were not used in the optimization, the equivalent plots are included in the appendix.

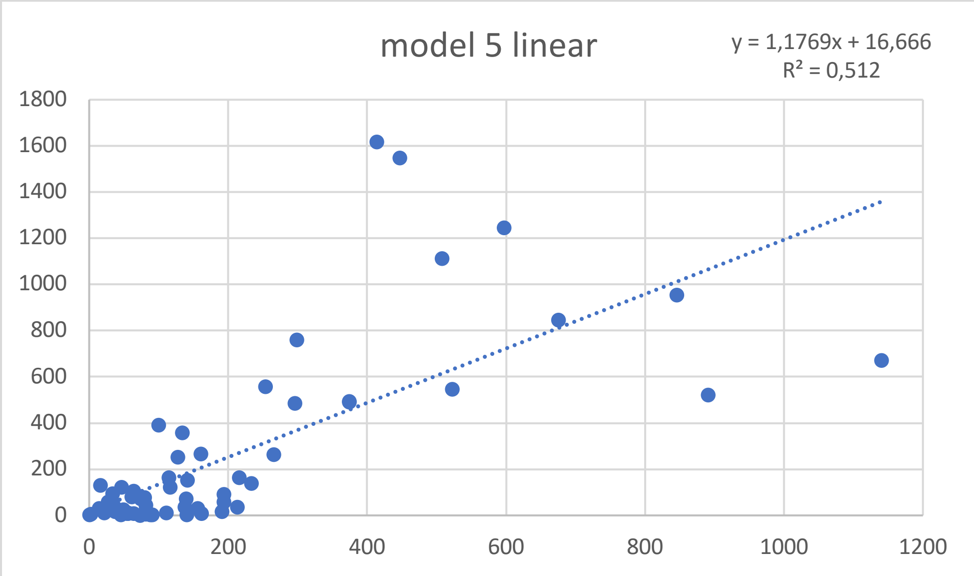

Figure 12: Comparison between expected and real values for model 5 on a linear scale

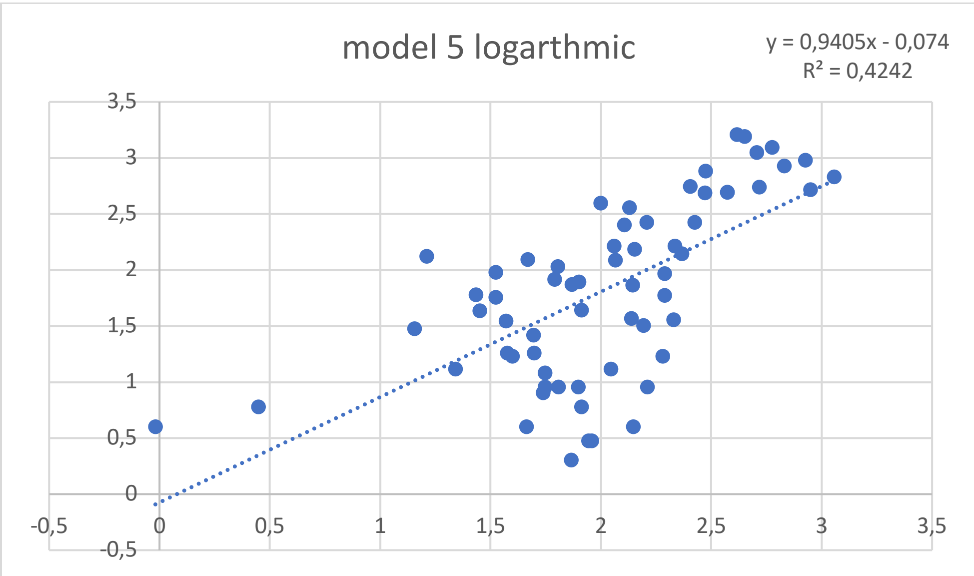

Figure 13: Comparison between expected and real values for model 5 on a logarithmic scale

As a result, it was concluded that, although model 5's indicators suggested better predictive performance, this model is strongly overestimating the demand for the smaller markets, and slightly overestimating for the bigger markets. Since the model is applicable to PSO routes, which are characterized by low demand, the decision that model 2 was the most suitable choice was made, due to the fact that the predicted values are closer to the actual values.

For the optimization model, these will be the demands to be fulfilled, for the two networks:

Table 18: Demand values for each O/D pair for the Rhodes network

Rhodes demand									
		Arrival							
		Astypalaia	Kalymnos	Karpathos	Kasos	Kastelorizo	Kos	Leros	Rhodes
Departure	Astypalaia		1	2	1	2	2	1	7
	Kalymnos	1		2	2	2	1	4	1
	Karpathos	2	2		11	2	1	2	57
	Kasos	1	2	7		1	2	2	16
	Kastelorizo	2	2	2	1		2	2	45
	Kos	2	4	2	2	2		2	4
	Leros	1	2	2	2	2	2		1
	Rhodes	4	1	57	19	41	29	1	

Table 19: Demand values for each O/D pair for the Thessaloniki network

Thessaloniki demand									
		Arrival							
		Chios	Ikaria	Kalamata	Kerkyra	Limnos	Samos	Skyros	Thessaloniki
Departure	Chios		2	8	1	7	3	2	179
	Ikaria	2		7	10	4	2	2	64
	Kalamata	8	7		1	8	9	5	128
	Kerkyra	11	10	2		7	1	5	2
	Limnos	5	4	8	7		4	2	63
	Samos	3	2	9	13	12		3	98
	Skyros	2	2	5	5	2	3		8
	Thessaloniki	187	88	144	1	56	110	14	

6. Analysis and Discussion

After running the optimization through the Fico Xpress software, satisfactory results were obtained in both networks. They will be presented and discussed in this chapter, one network at a time. As a reference, the calculation was performed in a Windows 10 Pro operating system, running in a computer with an Intel(R) Core(TM) i7-3770K CPU @ 3.50 GHz, and 8 GB of RAM memory.

6.1. Application to Rhodes Network

This network is considerably smaller than the network “based” in Thessaloniki (the current total costs of the Rhodes network are 42% of the value of the current total costs of the Thessaloniki network). At the beginning, with all the inputs for the model defined, “virtual constraints” were added, with the objective of accelerating the convergence towards the optimal solution. Examples of these virtual constraints are restricting the number of flights for each route (e.g. routes with no frequency imposition from the PSO or low demand, would have imposed a maximum of one frequency). Another virtual constraint that was attempted, was imposing that for every period of time, either no aircraft would depart the hub, or all the fleet would depart the hub, in an effort to promote the “hub effect”, and increase the number of passengers whose itinerary would be satisfied by connecting flights, reducing the total amount of flights.

Unfortunately, although this technique significantly reduced the computational times required for the solutions to be determined by the software package, after carefully analyzing the solutions provided, it was decided that these “virtual constraints” were not valid, because they were removing from the range of possible solutions, solutions which had lower total costs, besides following all the real constraints. Hence, the decision to remove these virtual constraints was taken, with the purpose of achieving the real optimal solution, at the expense of longer computational solving periods.

Another constraint that was not initially set, but after analyzing the result of the initial optimization calculations by the software was implemented, was the specification that each flight could not be repeated within less than 3 hours of another flight on the same route (i.e. a flight from A to B can only happen with more than 3 hours of interval from another flight from A to B). This had to be specified because the model was violating this condition in early solutions, and in reality having flights on the same route one immediately after the other only makes sense in very high demand routes (as is the example of the Lisbon – Porto city pair), not PSO routes.

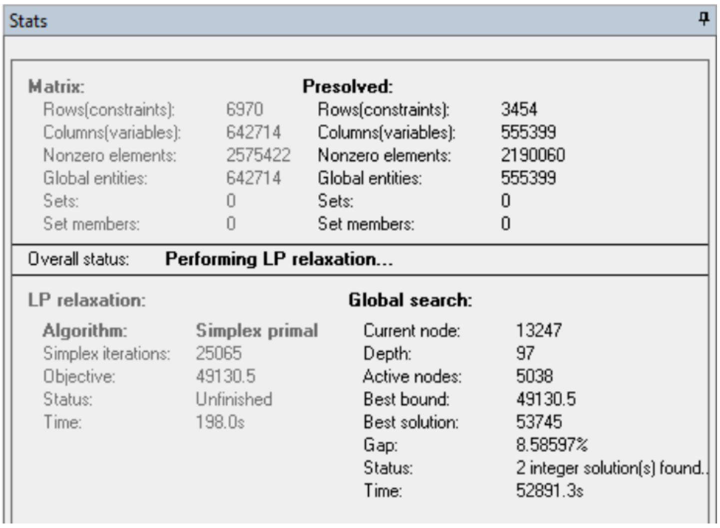

Figure 14: Key data provided by the optimization software, relative to the Rhodes network

This last constraint increased significantly the computational time (around 50%), but was seen as mandatory in order to present a plausible solution. The optimal solution for the Rhodes network was obtained after 14 hours and 41 minutes, with an optimality gap of 8.58%, as it can be seen from Figures 14 and 15.

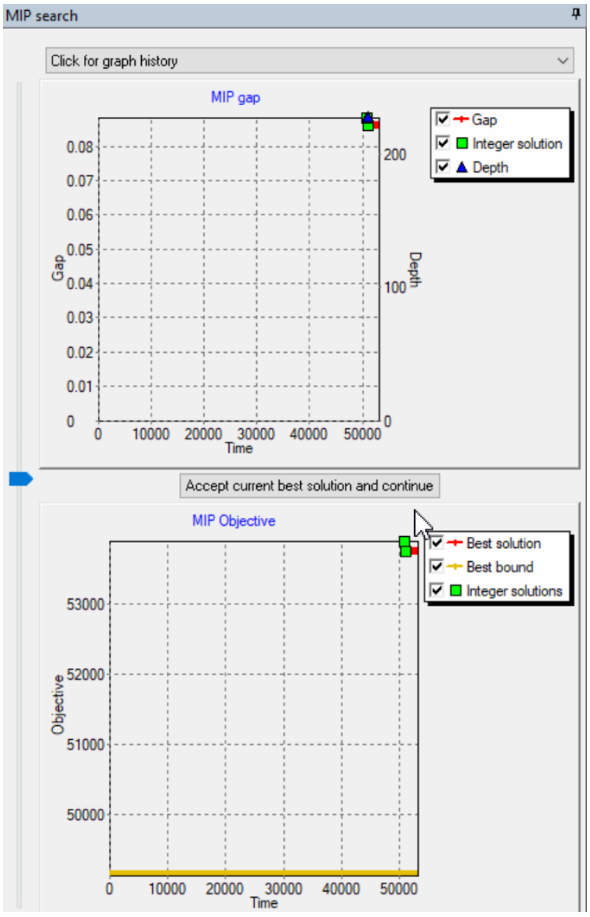

Figure 15: Plots presenting the evolution of the objective versus time, for the Rhodes network

The costs associated with this solution, and the comparison with the current network's costs are:

- **Flight operating costs**: 48 815€ for a total of 46 flights, compared to 53 771€ for the current network, which requires 50 flights, a reduction of 9.2% in cost;
- **Aircraft ground costs**: 75€ compared with 0€ for the current network. This increase is considered negligible when compared to the other reductions obtained by the model, especially having in mind that these 75€ apply for only one aircraft which exceeded the 2 hour limit for staying on the ground, for only 30 minutes, and this happened at the hub;
- **Passenger time costs**: 3 995€ for the time passengers spent on board, and 860€ for the time passengers spent waiting on the ground for a connecting flight, in a total of 4 855€ resulting in an average travel time of 1 hour and 17 minutes per passenger, compared with 6 470€ for the current network and an average travel time of 1 hour and 44 minutes per passenger. This means a reduction of 24.9% in cost;
- **Total cost of the network**: 53 745€ compared with 60 241€ for the current network. This means a reduction of 10.7% in the total cost of the network, with reductions in all the parameters, except a negligible increase from 0 to 75€ in the aircraft ground costs, fulfilling the objective of not only reducing the financial costs associated with the network, but also improving the quality of service provided to the passengers, through reduction of the door to door travel time.

The model produces a file presenting the results, with all the relevant information. For clarity, some results were deleted in order to show the whole range of content included in the file created by the software, in Figure 16.

```
The minimum cost is : 53745·
The flight operating costs are: 48815·
The aircraft ground costs are: 75·
The onboard time costs are: 3995·
The ground connection time costs are: 860·
there was one flight performed departing from 8 at time 1 arriving at 3 at time 3, the type was 1 carrying 33 passengers aboard
there was one flight performed departing from 8 at time 1 arriving at 6 at time 2, the type was 3 carrying 29 passengers aboard
there was one flight performed departing from 8 at time 2 arriving at 5 at time 4, the type was 2 carrying 33 passengers aboard
there was one flight performed departing from 3 at time 3 arriving at 4 at time 4, the type was 1 carrying 0 passengers aboard
there was one flight performed departing from 6 at time 3 arriving at 2 at time 4, the type was 3 carrying 1 passengers aboard
there was one flight performed departing from 2 at time 4 arriving at 7 at time 5, the type was 3 carrying 2 passengers aboard
there was one flight performed departing from 4 at time 4 arriving at 3 at time 5, the type was 1 carrying 2 passengers aboard
there was one flight performed departing from 5 at time 4 arriving at 8 at time 6, the type was 2 carrying 5 passengers aboard
there was one flight performed departing from 3 at time 5 arriving at 8 at time 7, the type was 1 carrying 5 passengers aboard
there was one flight performed departing from 7 at time 5 arriving at 1 at time 6, the type was 3 carrying 3 passengers aboard

there were 33 passengers transported, leaving from airport 8 at time 1 and arriving at airport 3 at time 3
there were 29 passengers transported, leaving from airport 8 at time 1 and arriving at airport 6 at time 2
there were 33 passengers transported, leaving from airport 8 at time 2 and arriving at airport 5 at time 4
there were 1 passengers transported, leaving from airport 7 at time 5 and arriving at airport 1 at time 6
there were 11 passengers transported, leaving from airport 3 at time 9 and arriving at airport 4 at time 10
there were 2 passengers transported, leaving from airport 6 at time 9 and arriving at airport 7 at time 10
there were 2 passengers transported, leaving from airport 7 at time 10 and arriving at airport 6 at time 11
there were 33 passengers transported, leaving from airport 3 at time 11 and arriving at airport 8 at time 13
there were 4 passengers transported, leaving from airport 6 at time 13 and arriving at airport 2 at time 14
there were 1 passengers transported, leaving from airport 1 at time 14 and arriving at airport 4 at time 16

there was 1 aircraft on the ground at airport 8 at time 1 and the type was 2
there was 1 aircraft on the ground at airport 6 at time 2 and the type was 3
there was 1 aircraft on the ground at airport 1 at time 6 and the type was 3
there was 1 aircraft on the ground at airport 8 at time 6 and the type was 2
there was 1 aircraft on the ground at airport 1 at time 7 and the type was 3
there was 1 aircraft on the ground at airport 8 at time 7 and the type was 1
there was 1 aircraft on the ground at airport 7 at time 9 and the type was 3
there was 1 aircraft on the ground at airport 7 at time 10 and the type was 3
there was 1 aircraft on the ground at airport 6 at time 11 and the type was 1

there were 1 passengers transported, leaving from airport 2 at time 4 with connection in airport 7 arriving there at time 5 and leaving at time 5 to airport 1, landing at time 6
there were 2 passengers transported, leaving from airport 5 at time 4 with connection in airport 8 arriving there at time 6 and leaving at time 7 to airport 3, landing at time 9
there were 2 passengers transported, leaving from airport 5 at time 4 with connection in airport 8 arriving there at time 6 and leaving at time 8 to airport 6, landing at time 9
there were 1 passengers transported, leaving from airport 3 at time 5 with connection in airport 8 arriving there at time 7 and leaving at time 8 to airport 6, landing at time 9

Aircraft type 1 payed for ground costs at airport 8 at time 30

there were 1 passengers transported, leaving from airport 6 at time 3 with connection in airports 2 arriving at time 4 and departing at time 4 to connect at airport 7 landing
there at time 5 and departing at time 5 arriving at airport 1 at time 6
there were 2 passengers transported, leaving from airport 4 at time 4 with connection in airports 3 arriving at time 5 and departing at time 5 to connect at airport 8 landing
there at time 7 and departing at time 8 arriving at airport 6 at time 9
there were 1 passengers transported, leaving from airport 5 at time 4 with connection in airports 8 arriving at time 6 and departing at time 7 to connect at airport 3 landing
there at time 9 and departing at time 9 arriving at airport 4 at time 10
there were 2 passengers transported, leaving from airport 3 at time 5 with connection in airports 8 arriving at time 7 and departing at time 8 to connect at airport 6 landing
there at time 9 and departing at time 9 arriving at airport 7 at time 10
```

Figure 16: Extract of the text file exported by the software with the details of the optimal solution

The movement of aircrafts and passengers in the network were compiled in a graph, visible in Figures 17, 18 and 19, where each line represents one of the 8 airports and each column represents one hour of the day. Each color represents one aircraft, and the numbers associated with each flight represent the passengers transported on board. It can be seen that there are 3 aircrafts operating. Horizontal lines represent an aircraft which is stopped on the ground in the airport associated with that line, while diagonal lines represent the flights. The airport is identified on the left of the line through its IATA (International Air Transport Association) code. For clarity, the association between the name of the airport and it's IATA code is shown in Table 20.

Table 20: Key for the airport IATA codes in the Rhodes network

IATA code	JTY	JKL	AOK	KSJ	KZS	KGS	LRS	RHO
Airport	Astypalaia	Kalymnos	Karpathos	Kasos	Kastelorizo	Kos	Leros	Rhodes

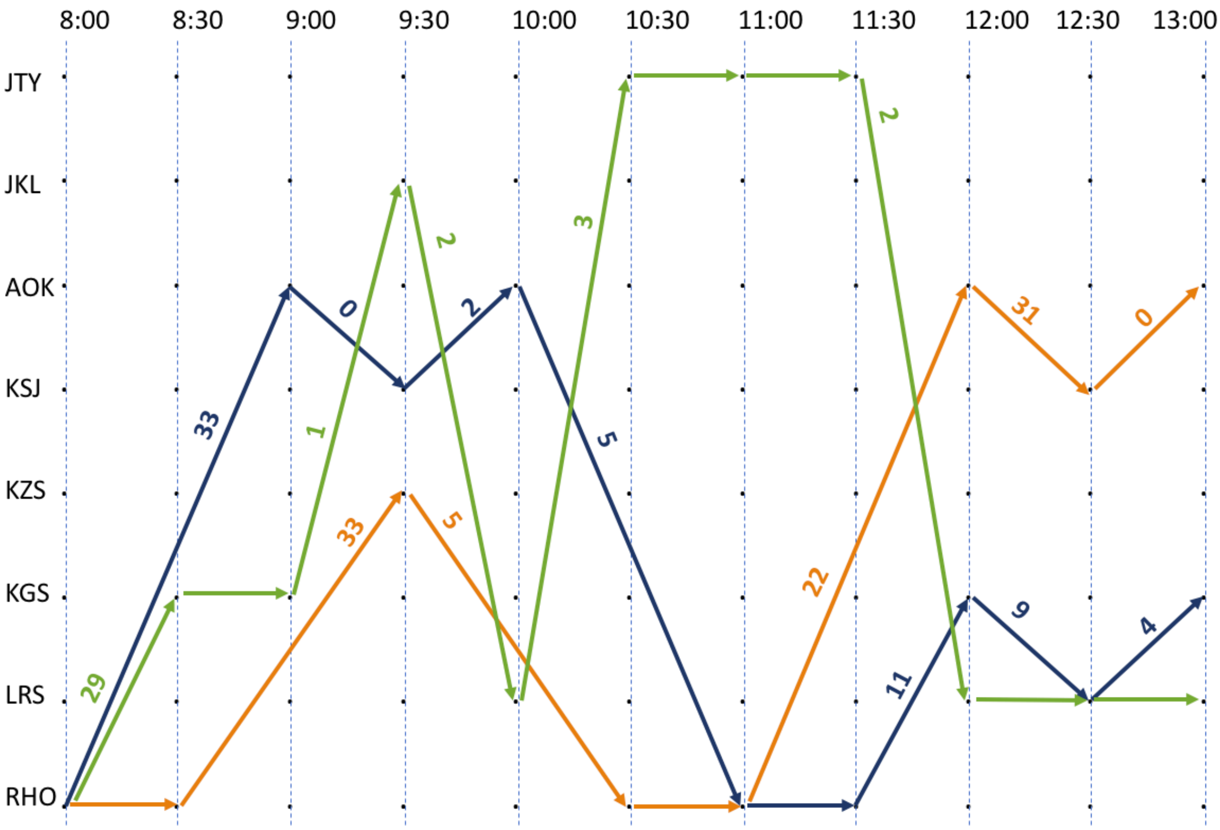

Figure 17: Representation of the flights from 08:00 to 13:00 in the Rhodes network

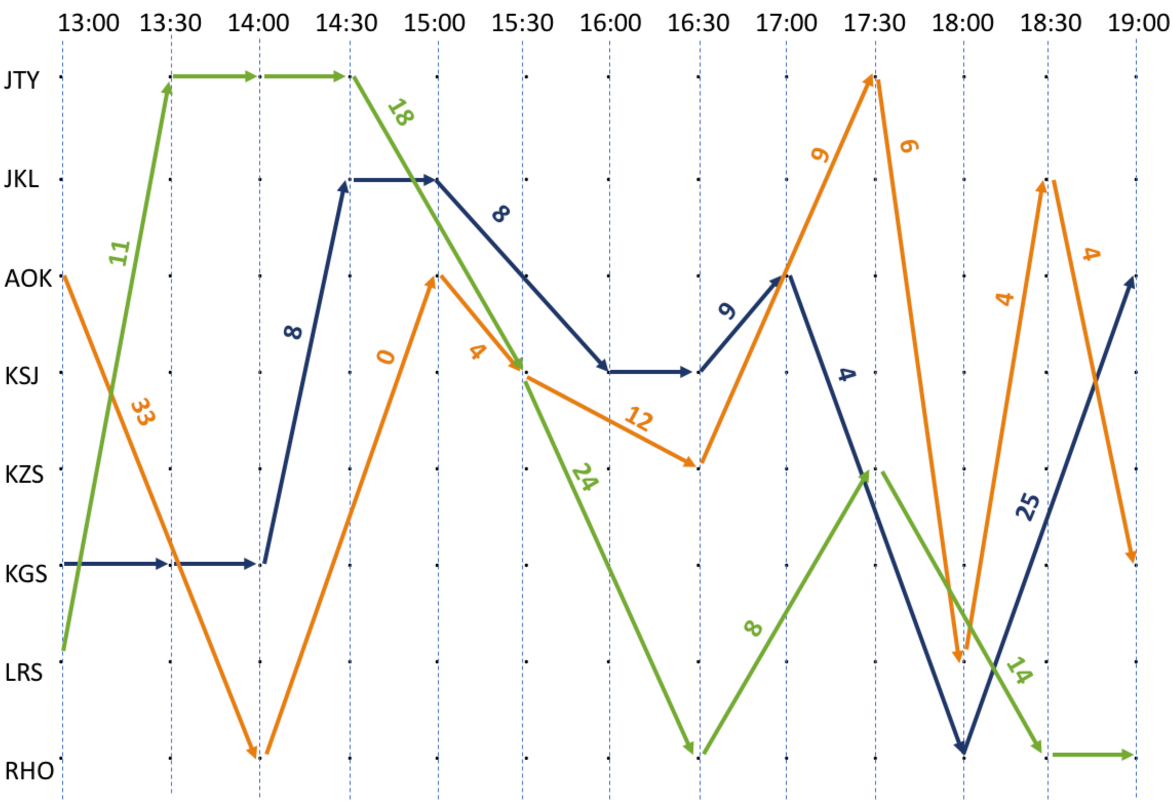

Figure 18: Representation of the flights from 13:00 to 19:00 in the Rhodes network

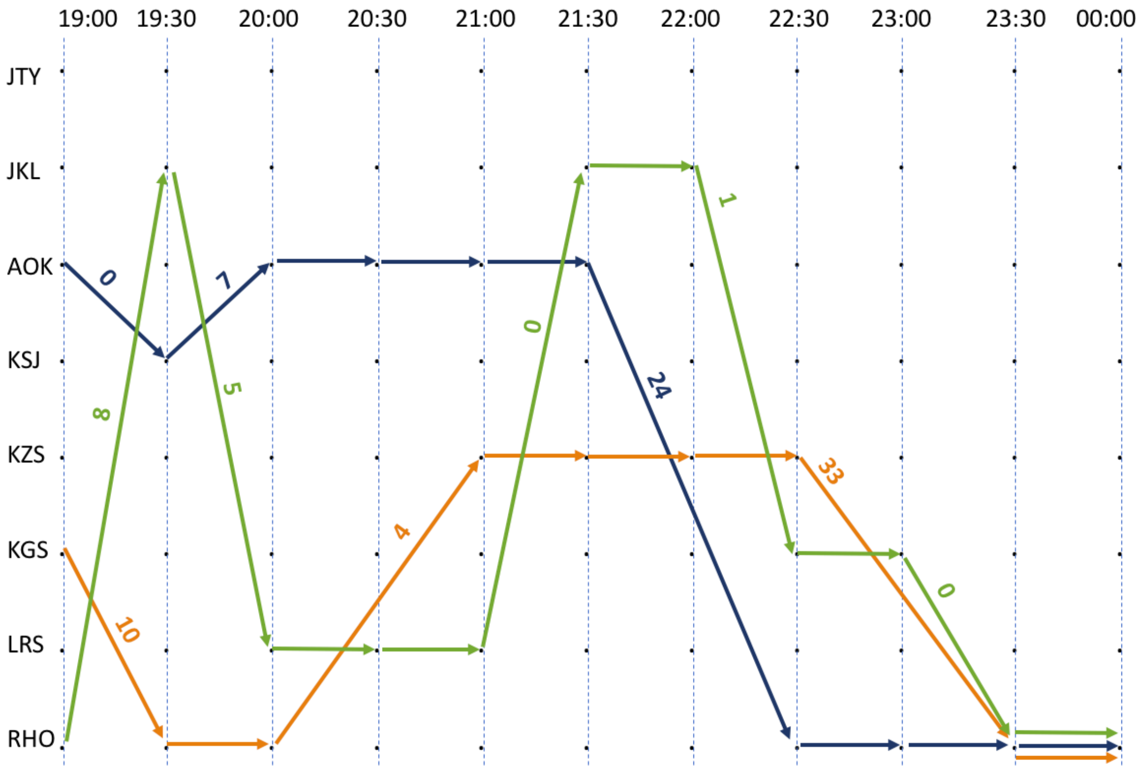

Figure 19: Representation of the flights from 19:00 to 00:00 in the Rhodes network

One of the immediate conclusions is the fact that there are several flights operated with zero passengers transported. This is a result of the nature of this network, inserted in a PSO scheme, as its main goal is not profitability, but assuring accessibility to these low demand regions, hence the need for government subsidies. The fleet used in this network was based on the real life fleet operating these routes and is comprised of 2 ATR42 aircraft (with 48 seats each) and 1 Dash 8 Q100 (with 37 seats).

6.2. Application to Thessaloniki Network

This network is, as already mentioned, significantly larger than the one discussed above. After realizing that applying the virtual constraints was actually excluding optimal and valid solutions from the solution domain, it was decided to run this optimization right from the start without applying the virtual constraints which had been removed during the optimization of the previous network. This decision was taken with the objective of guaranteeing that the software would consider every valid solution, at the expense of longer computational times. The optimal solution was found after 5 hours and 30 minutes, with an optimality gap of 11.26%, as it can be seen in Figure 20, alongside with more information. The model was then left running for another 15 hours, without successfully finding any other solution, as visible in Figure 21.

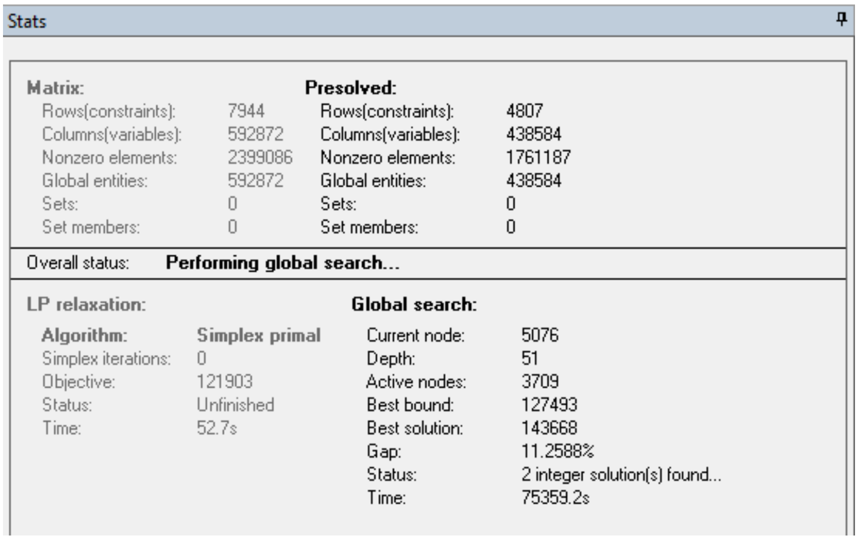

Figure 20: Key data provided by the optimization software, relative to the Thessaloniki network

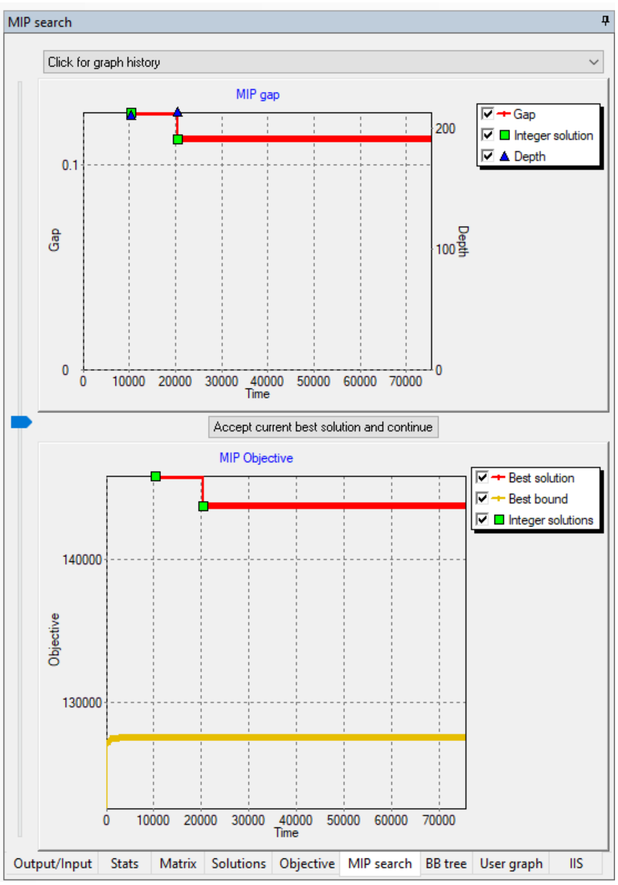

Figure 21: Plots presenting the evolution of the objective versus time, for the Thessaloniki network.

The costs associated with this solution, and the comparison with the current network's costs are:

- **Flight operating costs**: 117 978€ for a total of 54 flights, compared to 134 424€ for the current network, which requires 62 flights, a reduction of 12.2% in cost;
- **Aircraft ground costs**: 225€ compared with 0€ for the current network. This increase is considered negligible when compared to the other reductions obtained by the model, because as happened with the Rhodes network, this value is due to only one aircraft exceeding for 30 minutes the 2 hours on the ground, and this happened at Thessaloniki airport (considered the "hub");
- **Passenger time costs**: 21 900€ for the time passengers spent on board the aircraft, and 3 565€ for the time passengers spent waiting on the ground for a connecting flight, in a total of 25 465€ resulting in an average travel time of 1 hour and 52 minutes per passenger, compared with 27 637€ for the current network and an average travel time of 2 hours and 02 minutes per passenger. This means a decrease of 7.86% in cost;
- **Total cost of the network**: 143 668€ compared with 162 061€ for the current network. This means a reduction of 11.3% in the total cost of the network, having reduced once again both the direct financial costs to the airlines, as well as the time costs for the passengers. The explanation for the smaller improvements in this network's optimization, when compared to those obtained in Rhodes network's is thought to be related to the fact that, being a network with larger geographical distances and restrictions, there is a smaller margin for improvement. Moreover, the network's characteristics have a greater similarity to those of a normal (non-subsidized) network, when compared with Rhodes network's characteristics.

As with the previous network, an extract from the file obtained from the model with the results of the optimization and the graph demonstrating the flights in the network is presented in Figure 22.

```
The minimum cost is : 143668·
The flight operating costs are: 117978·
The aircraft ground costs are: 225·
The onboard time costs are: 21900·
The ground connection time costs are: 3565·
Aircraft type 3 payed for ground costs at airport 8 at time 1

there was one flight performed departing from 8 at time 1 arriving at 1 at time 4, the type was 4 carrying 70 passengers aboard
there was one flight performed departing from 8 at time 1 arriving at 3 at time 4, the type was 1 carrying 43 passengers aboard
there was one flight performed departing from 8 at time 1 arriving at 4 at time 3, the type was 2 carrying 1 passengers aboard
there was one flight performed departing from 8 at time 1 arriving at 6 at time 4, the type was 5 carrying 70 passengers aboard
there was one flight performed departing from 1 at time 4 arriving at 8 at time 7, the type was 4 carrying 70 passengers aboard
there was one flight performed departing from 6 at time 22 arriving at 2 at time 23, the type was 5 carrying 60 passengers aboard
there was one flight performed departing from 6 at time 22 arriving at 8 at time 25, the type was 1 carrying 32 passengers aboard
there was one flight performed departing from 8 at time 24 arriving at 3 at time 27, the type was 4 carrying 70 passengers aboard

there were 70 passengers transported, leaving from airport 8 at time 1 and arriving at airport 1 at time 4
there were 43 passengers transported, leaving from airport 8 at time 1 and arriving at airport 3 at time 4
there were 1 passengers transported, leaving from airport 8 at time 1 and arriving at airport 4 at time 3
there were 70 passengers transported, leaving from airport 8 at time 1 and arriving at airport 6 at time 4
there were 60 passengers transported, leaving from airport 1 at time 4 and arriving at airport 8 at time 7
there were 33 passengers transported, leaving from airport 1 at time 22 and arriving at airport 8 at time 25

there was 1 aircraft on the ground at airport 8 at time 31 and the type was 1
there was 1 aircraft on the ground at airport 8 at time 31 and the type was 3
there was 1 aircraft on the ground at airport 8 at time 31 and the type was 4
there was 1 aircraft on the ground at airport 8 at time 32 and the type was 1
there was 1 aircraft on the ground at airport 8 at time 32 and the type was 2
there was 1 aircraft on the ground at airport 8 at time 32 and the type was 3
there was 1 aircraft on the ground at airport 8 at time 33 and the type was 5

there were 8 passengers transported, leaving from airport 1 at time 4 with connection in airport 8 arriving there at time 7 and leaving at time 8 to airport 3, landing at time 11
there were 11 passengers transported, leaving from airport 4 at time 4 with connection in airport 8 arriving there at time 6 and leaving at time 7 to airport 1, landing at time 10
there were 2 passengers transported, leaving from airport 4 at time 4 with connection in airport 8 arriving there at time 6 and leaving at time 8 to airport 3, landing at time 11
there were 7 passengers transported, leaving from airport 4 at time 4 with connection in airport 8 arriving there at time 6 and leaving at time 7 to airport 5, landing at time 9
there were 1 passengers transported, leaving from airport 4 at time 4 with connection in airport 8 arriving there at time 6 and leaving at time 7 to airport 6, landing at time 10
there were 5 passengers transported, leaving from airport 4 at time 4 with connection in airport 8 arriving there at time 6 and leaving at time 7 to airport 7, landing at time 9
there were 9 passengers transported, leaving from airport 6 at time 4 with connection in airport 8 arriving there at time 7 and leaving at time 8 to airport 3, landing at time 11
there were 53 passengers transported, leaving from airport 8 at time 7 with connection in airport 5 arriving there at time 9 and leaving at time 9 to airport 2, landing at time 11

there were 2 passengers transported, leaving from airport 1 at time 4 with connection in airports 8 arriving at time 7 and departing at time 8 to connect at airport 3 landing there
at time 11 and departing at time 11 arriving at airport 7 at time 13
there were 10 passengers transported, leaving from airport 4 at time 4 with connection in airports 8 arriving at time 6 and departing at time 7 to connect at airport 5 landing there
at time 9 and departing at time 9 arriving at airport 2 at time 11
there were 7 passengers transported, leaving from airport 2 at time 11 with connection in airports 5 arriving at time 13 and departing at time 13 to connect at airport 8 landing there
at time 15 and departing at time 18 arriving at airport 3 at time 21
there were 7 passengers transported, leaving from airport 3 at time 11 with connection in airports 7 arriving at time 13 and departing at time 15 to connect at airport 5 landing there
at time 16 and departing at time 16 arriving at airport 2 at time 18
```

Figure 22: Extract of the text file exported by the software with the details of the optimal solution

Figures 23, 24 and 25 represent the flights of this network, and once again the airports are represented by their IATA codes. Table 21 relates the airport's name and its IATA code.

Table 21: Key for the IATA codes of the airports in the Thessaloniki network

IATA code	JKH	JIK	KLX	CFU	LXS	SMI	SKU	SKG
Airport	Chios	Ikaria	Kalamata	Kerkyra	Limnos	Samos	Skyros	Thessaloniki

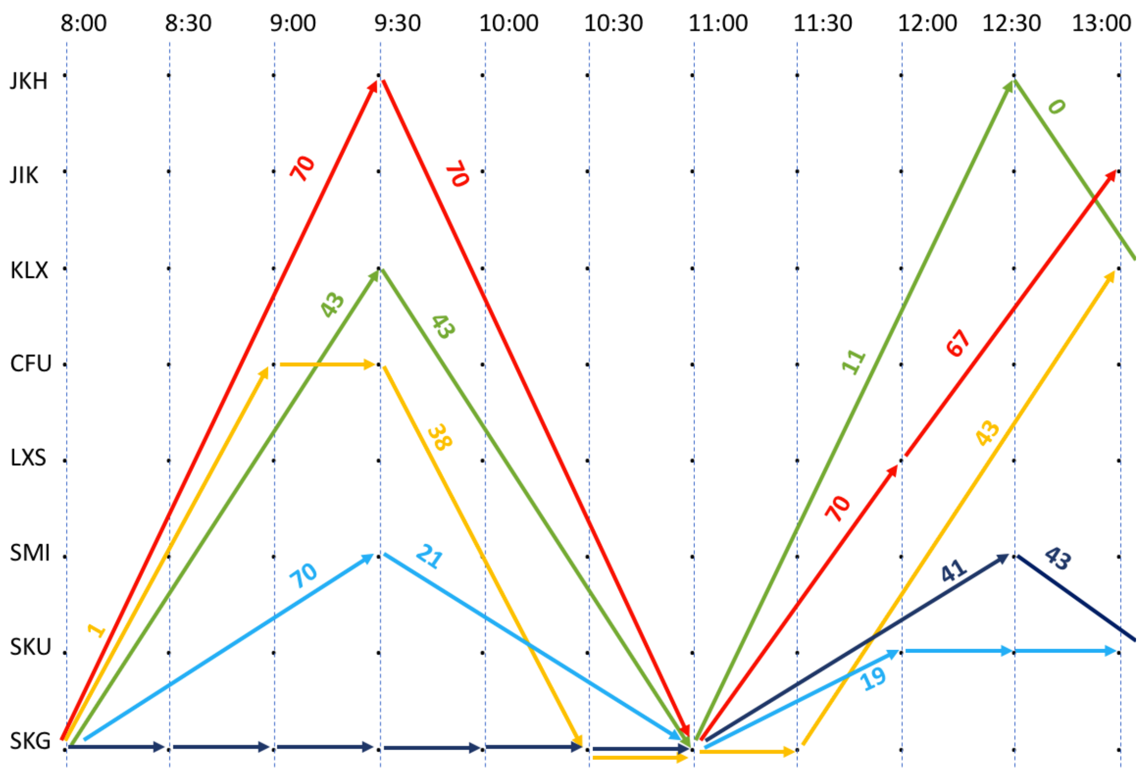

Figure 23: Representation of the flights from 08:00 to 13:00 in the Thessaloniki network

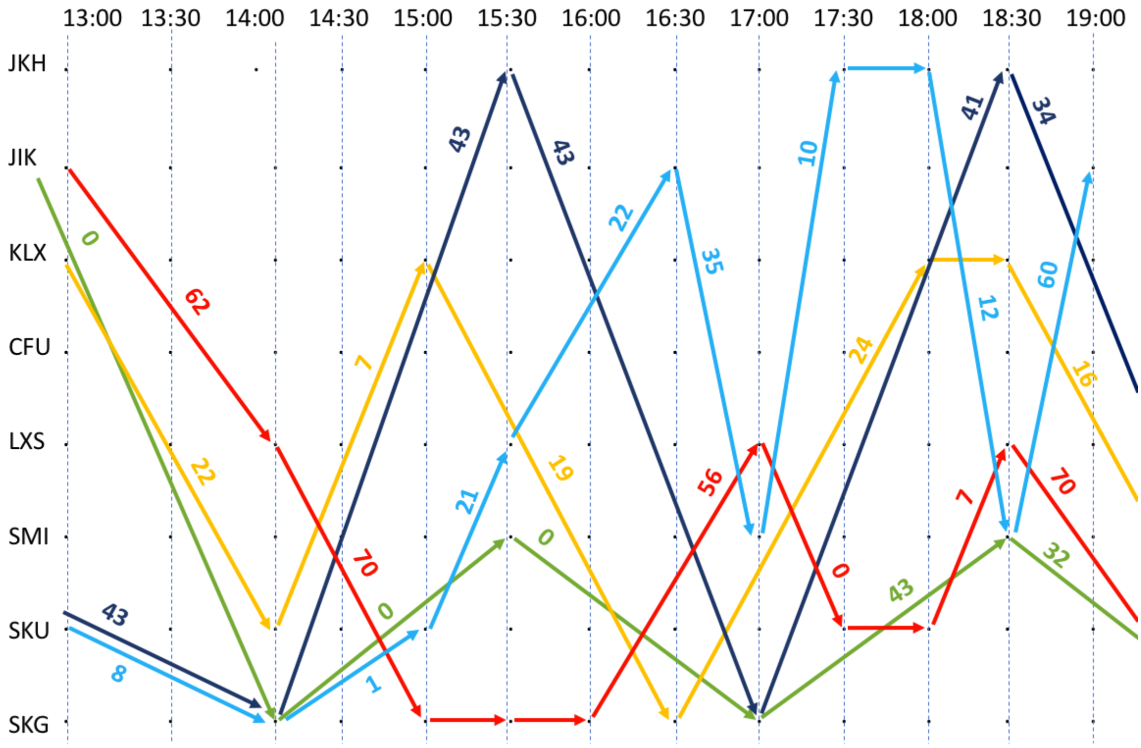

Figure 24: Representation of the flights from 13:00 to 19:00 in the Thessaloniki network

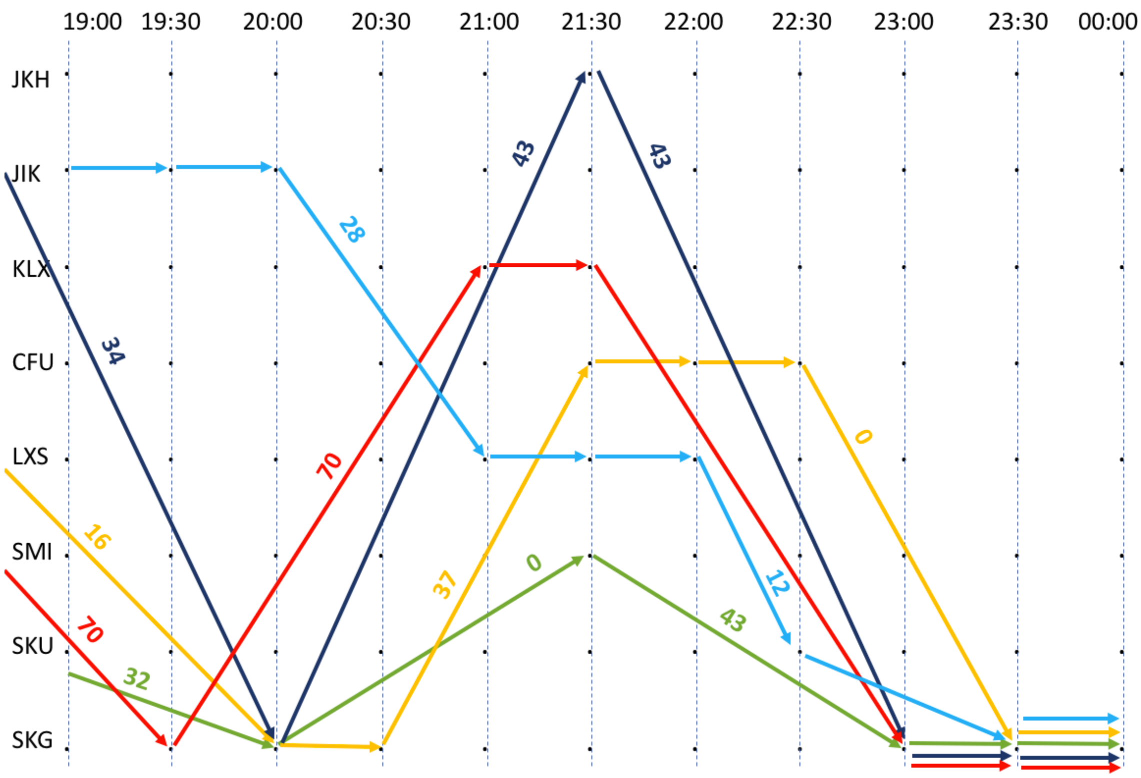

Figure 25: Representation of the flights from 19:00 to 00:00 in the Thessaloniki network

This network is, once again, clearly more complex than the previous network. Now there is a fleet of 5 aircraft in operation, comprised of 2 dash 8 Q100 (37 seats), 2 dash 8 Q400 (78 seats) and 1 ATR 42 (48 seats), based on real world information collected from the fleet operating these routes.

It is clear that, without any flight concentration constraint imposed mathematically, the model used the Thessaloniki airport as a hub, converging flights into and out of that airport at the same time, in order to increase the number of connecting passengers, reducing the overall number of flights required. Besides this, due to the significant imposition of flights by the PSO, there are still some flights being operated without passengers allocated to them. Nevertheless, these flights still have to be operated, to satisfy the public service requirements imposed (and financed) by the PSO.

6.3. Exploring Scenarios

As mentioned before, there were two demand model candidates considered statistically viable in chapter 5, but model 2 was preferred to model 5, due to the fact that it was considered that model 5, although performing better in terms of KPI's, with better values in LL, or AIC was overestimating the demand too significantly, which led the decision to use model 2 as the final demand model. With the results obtained by model 2 already presented above, the demand values provided by model 5 were used in order to perform a sensitivity analysis, and verify if the improvement in the results varied significantly with this new demand. The optimization was run with the new demand for the Rhodes and Thessaloniki networks and the results will be presented next.

6.3.1. Rhodes Network

- **Flight operating costs**: 48 815€ for a total of 47 flights, compared to 53 771€ for the current network, which requires 50 flights, a reduction of 9.2% in cost. Interestingly, this is the same value of flight operating costs for the optimization that was performed with the previous demand model. It is believed this is explained by the fact that since this network has such a low demand, and a comparatively high amount of imposed flights (by the PSO), the model does not need to exceed the imposed flights unless demand increases significantly;
- **Aircraft ground costs**: 0€ compared with 0€ for the current network. This means that in this optimization, the model was able to avoid leaving an aircraft on the ground for more than 2 hours, compared to the previous calculation (with the main demand model), where the solution implied an aircraft staying for 2 hours and 30 minutes on the ground;
- **Passenger time costs**: 6 820€ for the time passengers spent on board the aircraft, and 2 465€ for the time passengers spent waiting on the ground for a connecting flight, in a total of 9 285€ resulting in an average travel time of 1 hour and 42 minutes per passenger, compared with 19 042€ for the current network and an average travel time of 3 hours and 30 minutes per passenger. These costs were calculated for the new values of demand, as a higher demand for passengers will imply higher overall social costs. This optimization obtained a reduction of

51.2% in cost. This value is obviously very significant, and the proposed explanation for such a high value is the demand model imposing a demand too high for the current network, which is not prepared for it, leading to long waiting times in routes that are being considered as having high demand;

- **Total cost of the network**: 58 100€ compared with 72 813€ for the current network. This means a reduction of 20.2% in the total cost of the network, with reductions in all the parameters. This higher improvement of the optimized network when compared to the previous demand model, is obviously driven by the significant reduction of the passenger time costs, but validates the quality of the results obtained by the previous optimization, whose demand input is expected to be more exact.

6.3.2. Thessaloniki network:

- **Flight operating costs**: 138 815€ for a total of 67 flights, compared to 134 424€ for the current network, which requires 62 flights, an increase of 3% in cost. This result is obviously not satisfactory, but it is expected that with such an increase in demand (given by this alternative demand model), from 1357 to 1861 passengers in the network (an increase of 37%), would require more flights being performed in the "current" network for the comparison to be fair, which was not considered, due to the fact that there is no reliable way of estimating this increase in flights. Hence, this remark is made, and the current cost obtained by real values from the network is considered;
- **Aircraft ground costs**: 75€ compared with 0€ for the current network. As it was commented on the previous demand model, this increase is considered negligible when compared to the other reductions obtained by the model and this value is due to only one aircraft exceeding for 30 minutes the 2 hours on the ground, and this happened at Thessaloniki airport (considered the "hub"), hence this slight increase is not considered relevant to the results;
- **Passenger time costs**: 31 855€ for the time passengers spent on board the aircraft, and 9 265€ for the time passengers spent waiting on the ground for a connecting flight, in a total of 41 120€ resulting in an average travel time of 2 hours and 12 minutes per passenger, compared with 53 155€ for the current network and an average travel time of 2 hours and 51 minutes per passenger. This means a decrease of 22.6% in cost;
- **Total cost of the network**: 180 010€ compared with 187 579€ for the current network. This means a reduction of 4% in the total cost of the network, having increased direct financial costs to the airlines, but reduced the time costs for the passengers. The key driver for the weaker performance of this optimization compared to the previous demand was the fact that the flight operating costs increased significantly on these optimized networks (an increase of 17.7% between demand models), and the flight operating costs of the current network were kept constant.

Nevertheless, this result is thought to validate the quality of the results presented above, proving that even though running significantly different demand models, the optimization model was always able to present significant improvements (ranging from 6.8 to 28.7%) in the total cost of the network, reducing always both the direct cost to the airlines and the indirect time costs for the passengers.

6.3.3. Different Scenarios

Besides the results already presented in this chapter, different scenarios were considered during this thesis, in an attempt to achieve better results. These results will be briefly discussed in the following paragraphs.

One of the attempts was to re-design the network, in terms of the impositions by the PSO. Hence, different configurations were attempted in the optimization model, in order to compare the obtained result with the best result already achieved for that particular network. To do this, by looking at the demand values obtained, and to the geographical position of the different islands, new possible configurations were attempted, as well as one network where the PSO imposition was that every island had to have a direct flight to its network's hub. Unfortunately, none of these attempted networks resulted in a better overall cost, which led them to be discarded.

Furthermore, it was attempted to divide the optimization into two different days. The first day, would have flights imposed for all of the routes which have a PSO in force, but would have less frequencies in the busier routes. The remaining flights would be fulfilled in the second day, which would only have these remaining frequencies imposed. The reasoning behind this attempt was to "relieve" the network from so many flights for certain routes, which are flown with no passengers assigned, in the result of the optimization. This attempt also resulted in a higher total cost and ended up being discarded as well. Nevertheless, one interesting conclusion from this attempt was that, in the second day, the software only took 11 seconds to reach the optimal solution, although the network still had 8 airports and 33 time periods. This demonstrates that the reason behind the long computational times comes from the complex demand, frequencies and seat number impositions from the full network.

7. Conclusions and Future Research

In this chapter, the conclusions of the research will be presented, followed by an analysis of its limitations and a suggestion of future work.

7.1. Conclusions

The present work adapted a published optimization model ([18], [19]), in order to apply it to two case studies situated in the Greek PSO network. It demonstrated that the network can be improved, not only from a financial point of view, but also regarding passenger level of service.

One of the main strengths of this work is combining both the development of a predictive model and a flight scheduling and fleet optimization model. The predictive model was developed following all the commonly accepted practices (using as references the published literature such as [21] and [22]), in order to maximize the quality of the demand prediction. The dataset presented challenges for the development of the GZLM, such as significant overdispersion of data, which led to the rejected attempt to use a Negative binomial regression, followed by a Poisson regression with a Pearson chi-square scale parameter method. This resulted in the inability to use as many explanatory variables as initially intended, due to their statistical non-significance. These variables were suggested as a result from a comprehensive literature review and analysis into the particularities of the Greek market. Nevertheless, two demand models complied with the desired level of performance in the evaluated KPI's, and were tested in the optimization model.

With the predictive model developed, the case studies were presented, and the basis for comparison was defined, by quantifying the costs of the current network. The two case studies represent, within the same country, networks with different characteristics, in terms of number of passengers carried, size of operating fleet and geographical distances. This was done with the objective of allowing a broader characterization of the Greek market.

The optimization resulted in significant improvements, in the order of 10% in both networks, while following all the constraints specified, in order to properly characterize the particularities of the Greek PSO market. The process faced several challenges in its implementation, particularly the considerable complexity of the problem under analysis, in terms of amount of routes, fleet size and amount of time periods. These challenges meant that a significant amount of effort had to be put into improving the mathematical formulation, achieving a much simpler but less intuitive formulation which allowed for the solving of the problem by a consumer level computer. Also, a significant amount of attention was put into imposing virtual constraints and pre-processing of data, which accelerated the solving process, while not excluding the optimal solutions from the scope of analysis.

7.2. Limitations

This research holds some limitations, which should be acknowledged by the readers. Firstly, regarding the results of the predictive model, its inherent uncertainty should not be disregarded. Every predictive model has an associated uncertainty [25] and this dataset, with challenging characteristics, will not be an exception. This is partly explained by the lack of data available and by the low demand values for these O/D pairs and is demonstrated by the significant difference in the prediction between the two final candidates for demand model.

Moreover, regarding the optimization model, although 48 hours is an acceptable duration for running this computation, required twice a year, its convergence towards optimality is limited. This is demonstrated by the difficulty for the model to improve solutions. After finding the first solution, and running for three times that duration, no additional improvements were achieved. Also, the optimality gap of the calculations was around 8% which is not ideal, although significant attention was put into the pre-processing and virtual constraints, in order to improve such situation.

7.3. Future Work

This research opens several opportunities for future research, which will be discussed in the following paragraphs.

From the point of view of the predictive model, only data from 240 O/D pairs was analyzed. In Greece, there are at least 40 airports operating commercially at the time of this research, which means there are 1560 possible O/D pairs for a comprehensive predictive model of the Greek market. Hence, there are several research opportunities to expand the scope of this model. Also, there is an opportunity to develop a research in partnership with Greek authorities that could facilitate the required data for it, which would integrate more explanatory variables. This could make the GDP/capita a statistically significant explanatory variable, if data from each specific island could be obtained. More attention into attempting to obtain significance from variables related to the importance of tourism and of the competition of the ferry boat service is also an important opportunity.

Regarding the optimization model, there are several research opportunities to build on the present work. One interesting opportunity, is to develop a similar research, but optimizing the network for the winter months, using demand data from that period. Then, an in depth comparison between both networks should bring interesting data, in a market with such a strong effect from seasonality, mostly due to tourism. Another interesting opportunity is to perform a similar research, but involving a bigger network, up to all the Greek market, with 40 airports, and taking into account all the PSO impositions and constraints describing the whole market.

Another interesting opportunity is to perform a follow-up analysis of this research, just like Antunes et al. [41] performed, on the 2013 paper [18], working in close relationship with the airline operating the PSO network in the Azores. This could better quantify the real costs associated with the operation, and analyze in depth how could a previous period of operation be improved, from where all the data was already processed and available.

The last suggestion of further research based on this work is to perform a more comprehensive analysis, including the effect of ticket pricing on demand. This is a complex iterative analysis, because price affects demand, and vice-versa. Besides this, because the PSO networks have prices imposed, there is less freedom for the airline in terms of pricing, which could reduce the complexity of this analysis, only having to consider pricing in the routes not belonging to the PSO network.

8. References

[1] ACI Europe, "The Social and Economic Impact of Airports," York Aviation, Geneva, 2004.

[2] Halpern, N. and Bråthen, S., "Impact of airports on regional accessibility and social development," *Journal of Transport Geography,* pp. 1145-1154, 2011.

[3] Lieshout, R., "Measuring the size of an airport's catchment area," *Jornal of Transport Geography,* vol. 25, pp. 27-34, 2012.

[4] Lohmann, G., Albers, S., Koch, B., Pavlovich, K., "From hub to tourist destination – An explorative study of Singapore and Dubai's aviation-based transformation," *Journal of Air Transport Management,* vol. 15, pp. 205-211, 2009.

[5] Zou, L., Chen, X., "The effect of code-sharing alliances on airline profitability," *Journal of Air Transport Management,* vol. 58, pp. 50-57, 2017.

[6] Raynes, C., Tsui, K.W.H., "Review of Airline-within-Airline strategy: Case studies of the Singapore Airlines Group and Qantas Group," *Case Studies on Transport Policy,* vol. 7, pp. 150-165, 2019.

[7] Merkert, R., Swidan, H., "Flying with(out) a safety net: Financial hedging in the airline industry," *Transportation Research Part E,* vol. 127, 2019.

[8] European Commission, [Online]. Available: https://ec.europa.eu/transport/modes/air/internal-market/pso_en. [Accessed 12 06 2019].

[9] European Commission, [Online]. Available: https://publications.europa.eu/en/publication-detail/-/publication/76f866ac-ee1d-4981-83b2-8ee76a85710f/language-en. [Accessed 27 08 2019].

[10] European Commission, "PSO Inventory Table," [Online]. Available: https://ec.europa.eu/transport/sites/transport/files/pso_inventory_table.pdf. [Accessed 27 08 2019].

[11] Wittman, M.D., Allroggen, F., Malina, R., "Public service obligations for air transport in the United States and Europe: Connectivity effects and value for money," *Transportation Research Part A,* vol. 94, pp. 112-128, 2016.

[12] European regions airline association , "Public Service Obligation (PSO) routes in Europe," 2016.

[13] Williams, G., Pagliari, R., "A comparative analysis of the application and use of public service obligations in air transport within the EU," *Transport Policy,* vol. 11, pp. 55-66, 2004.

[14] U.S Department of Transportation, "Essential Air Service," [Online]. Available: https://www.transportation.gov/policy/aviation-policy/small-community-rural-air-service/essential-air-service. [Accessed 28 08 2019].

[15] Australian Department of Infrastructure, Transport, Cities and Regional development, "Remote Air Services Subsidy Scheme," [Online]. Available: https://www.infrastructure.gov.au/aviation/regional/rass.aspx. [Accessed 28 08 2019].

[16] Santana, I., "Do Public Service Obligations hamper the cost competitiveness of regional airlines?," *Journal of Air Transport Management,* vol. 15, pp. 344-349, 2009.

[17] Skreikes, M., "Public Service Obligations: the European perspective," *3rd International Forum on Air Transport in Remoter Regions,* 2003.

[18] Pita, J.P., Antunes, A.P., Barnhart, C., Menezes, A.G., "Setting public service obligations in low-demand air transportation networks: Application to the Azores," *Transportation Research Part A,* vol. 54, pp. 35-48, 2013.

[19] Pita, J.P, Adler, N., Antunes, A.P., "Socially-oriented flight scheduling and fleet assignment model with an application to Norway," *Transportation Research Part B,* vol. 61, pp. 17-32, 2014.

[20] Lee, Y., Alam, M.M., Noh, M., Rönnegård, L., Skarin, A., "Spatial modeling of data with excessive zeros applied to reindeer pellet-group counts," *Ecology and Evolution,* vol. 6, p. 7047–7056, 2016.

[21] McCullagh, P., Nelder, J., Generalized Linear Models, Boca Raton: Chapman and Hall/CRC, 1989.

[22] Turkman, M.A.A., Silva, G.L., Generalized Linear Models, Peniche: SPE Editors, Portuguese Statistical Society, 2000.

[23] Bradley, S.P., Hax, A.C., Magnanti, T.L., Applied mathematical programming, Addison-Wesley Publishing Company, 1977.

[24] Nemhauser, G., Wolsey, L., Integer and combinatorial optimization, Wiley-Interscience, 1988.

[25] Neufville, R., Odoni, A., Airport Systems planning, design and management, McGraw-Hill Education, 2003.

[26] Washington, S.P., Karlaftis, M.G., Mannering, F.L., Statistical and Econometric Methods for Transportation Data Analysis, CRC Press, 2010.

[27] Jorge-Calderón, J.D., "A demand model for scheduled airline services on international European routes," *Journal of Air Transport Management,* vol. 3, pp. 23-35, 1997.

[28] Grosche, T., Rothlauf, F., Heinzl, A., "Gravity models for airline passenger volume estimation," *Journal of Air Transport Management,* vol. 13, pp. 175-183, 2007.

[29] Wadud, Z., "Modeling and Forecasting Passenger Demand for a New Domestic Airport with Limited Data," *Transportation Research Record,* pp. 59-68, 2011.

[30] Barnhart, C., Fearing, D., Odoni, A., Vaze, V., "Demand and capacity management in air Transportation," *EURO Journal on Transportation and Logistics,* vol. 1, pp. 135-155, 2012.

[31] Demirsoy, Ç., "Analysis of stimulated domestic air transport demand in turkey," 2012.

[32] Adeniran, A.O., Adeniran, A.A., "Econometric Modeling of Passenger Demand for International Air Transport in Nigeria Airports," *American Journal of Traffic and Transportation Engineering,* vol. 2, pp. 39-44, 2017.

[33] Carmona-Benítez, R.B., Nieto, M.R., Miranda, D., "An Econometric Dynamic Model to estimate passenger demand for air transport industry," *Transportation Research Procedia,* vol. 25, pp. 17-29, 2017.

[34] Kluge, U., Cook, A.J., Paul, A., Cristobal, S., "Factors influencing European passenger demand for air transport," in *Air Transport Research Society World Conference*, 2017.

[35] Devoto, R., Farci, C., Liliu, F., "Analysis and forecast of air transport demand in Sardinia's airports as a function of tourism variables," *Urban Transport VIII,* vol. 60, 2002.

[36] Erjongmanee, S., Kongsamutr, N., "Air Passenger Estimation Using Gravity Model and Learning Approaches: Case Study of Thailand," in *5th International Conference on Advanced Informatics: Concept Theory and Applications (ICAICTA)*, Thailand, 2018.

[37] Lohatepanont, M., Barnhart, C., "Airline Schedule Planning: Integrated Models and Algorithms for Schedule Design and Fleet Assignment," *TRANSPORTATION SCIENCE,* vol. 38, pp. 19-32, 2004.

[38] Sherali, H.D., Bae, K.H., Haouari, M., "Integrated Airline Schedule Design and Fleet Assignment: Polyhedral Analysis and Benders' Decomposition Approach," *INFORMS Journal on Computing,* vol. 22, pp. 500-513, 2010.

[39] Jamili, A., "A robust mathematical model and heuristic algorithms for integrated aircraft routing and scheduling, with consideration of fleet assignment problem," *Journal of Air Transport Management,* vol. 58, pp. 21-30, 2016.

[40] Liu, T.K., Jeng, C.R., Chang, Y.H., "Disruption Management of an Inequality-Based Multi-Fleet Airline Schedule by a Multi-Objective Genetic Algorithm," *Transportation Planning and Technology,* vol. 31, pp. 613-639, 2008.

[41] Antunes, A.P., Santos, M.G., Pita, J.P., Menezes, A.G., "Study on the evolution of the air transport network of the Azores," *Transportation Research Part A,* vol. 118, pp. 837-851, 2018.

[42] Iliopoulou, C., Kepaptsoglou, K., Karlaftis, M.G., "Route planning for a seaplane service: The case of the Greek Islands," *Computers&OperationsResearch,* vol. 59, pp. 66-77, 2015.

[43] Dožić, S., Kalić, M., "Three-stage airline fleet planning model," *Journal of Air Transport Management,* vol. 46, pp. 30-39, 2015.

[44] Ma, Q., Song, H., Zhu, W., "Low-carbon airline fleet assignment: A compromise approach," *Journal of Air Transport Management,* vol. 68, pp. 86-102, 2017.

[45] Eurocontrol, Standard Inputs for EUROCONTROL Cost-Benefit Analyses," 8th edition, Brussels, 2018.

[46] Domencich, T.A., Mcfadden, D., Urban Travel Demand: A Behavioral Analysis., North-Holland Publishing Co., 1975.

9. Appendix

9.1. Plots of predicted vs real demand

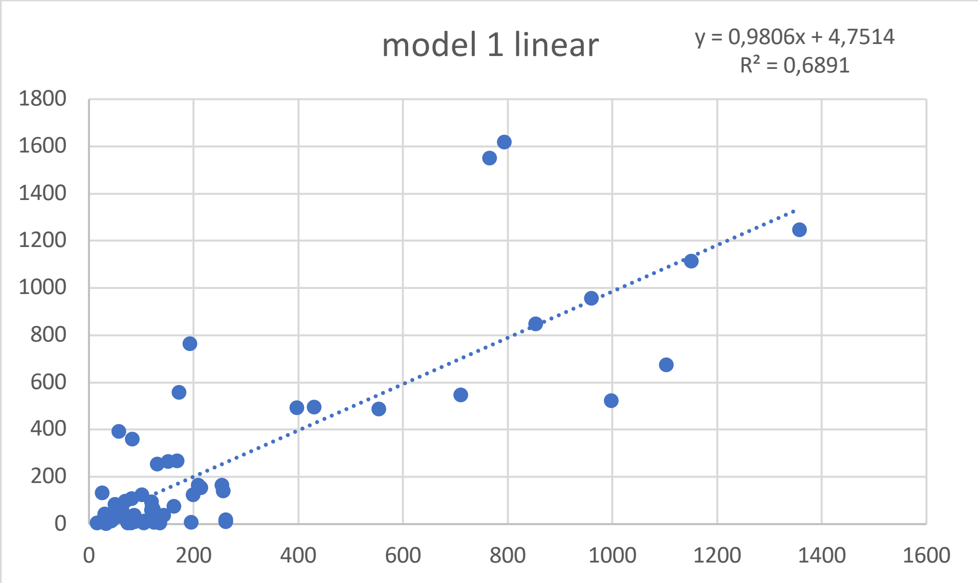

Figure 26: Comparison between expected and real values for model 1 on a linear scale

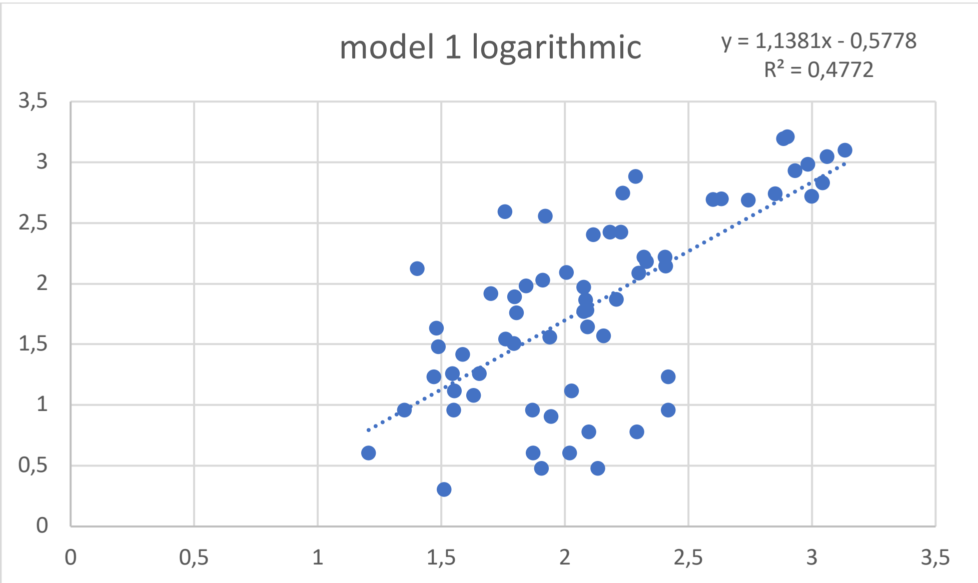

Figure 27: Comparison between expected and real values for model 1 on a logarithmic scale

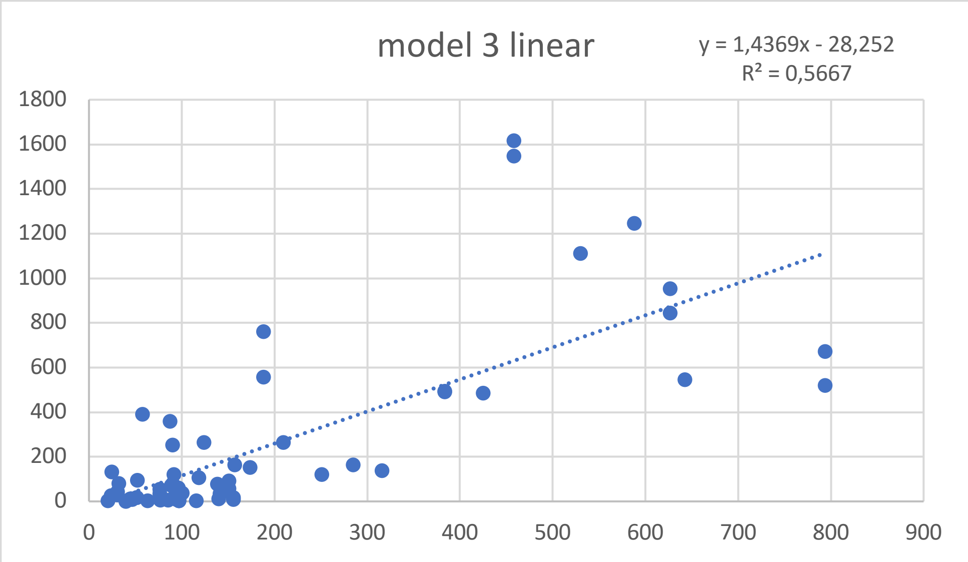

Figure 28: Comparison between expected and real values for model 3 on a linear scale

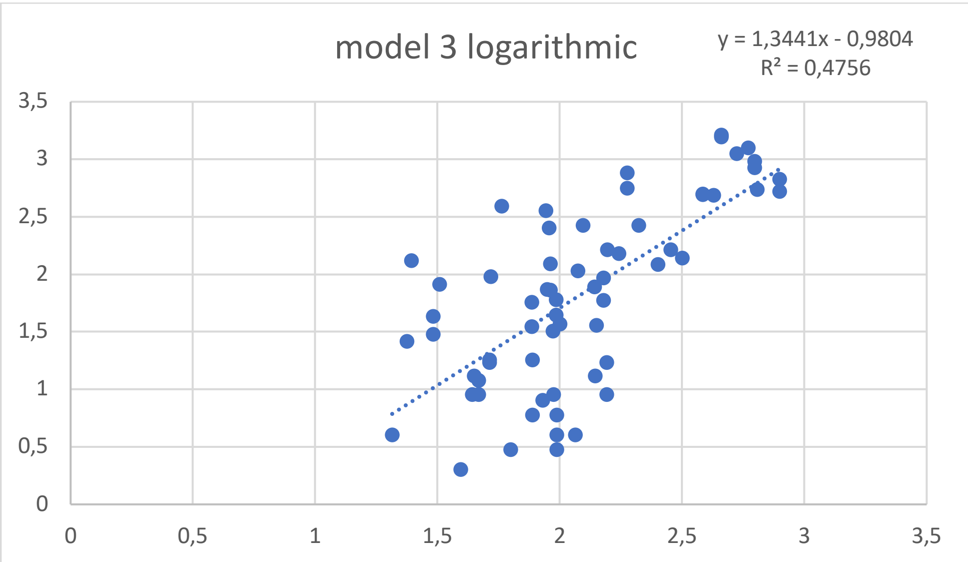

Figure 29: Comparison between expected and real values for model 3 on a logarithmic scale

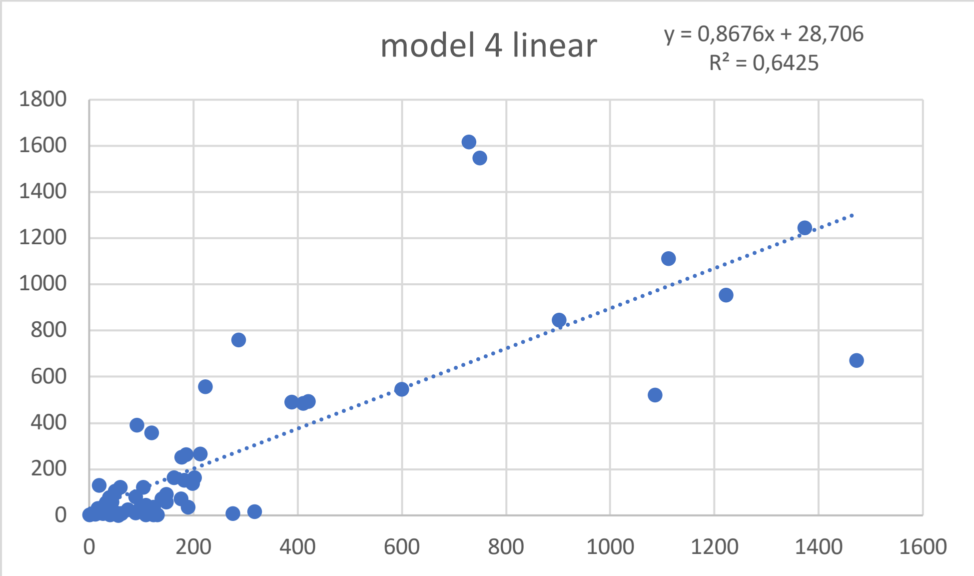

Figure 30: Comparison between expected and real values for model 4 on a linear scale

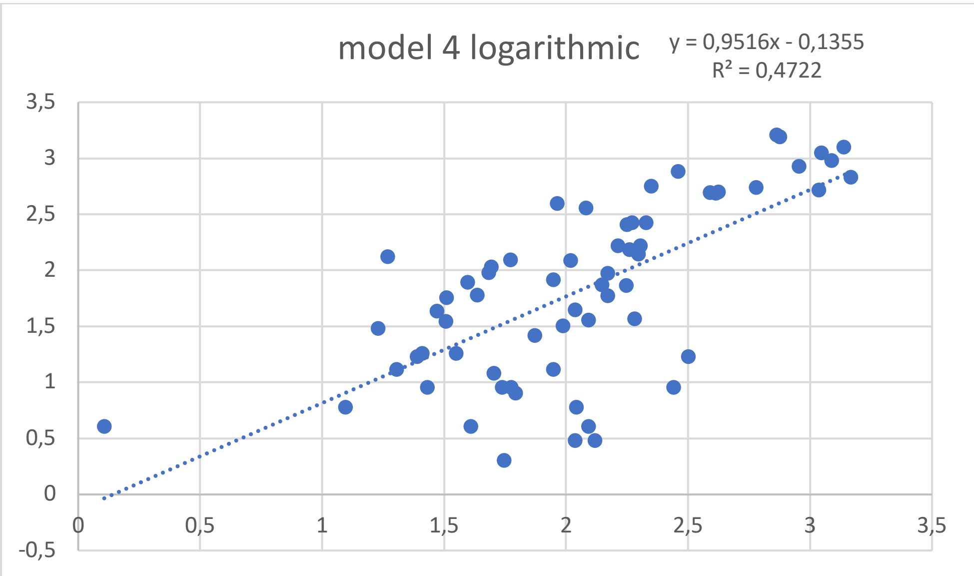

Figure 31: Comparison between expected and real values for model 4 on a logarithmic scale